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### Uncertainty and the Politics of Employment Protection

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## Abstract

This paper investigates the role that idiosyncratic uncertainty plays in shaping social preferences over the degree of labor market flexibility, in a general equilibrium model of dynamic labor demand where the productivity of firms evolves over time as a Geometric Brownian motion. A key result demonstrated is that how the economy responds to shocks, i.e. unexpected changes in the drift and standard deviation of the stochastic process describing the dynamics of productivity, depends on the power of labor to extract rents and on the status quo level of firing costs. In particular, we show that when firing costs are relatively low to begin with, a transition to a rigid labor market is favored by all and only the employed workers with idiosyncratic productivity below some threshold value. A more volatile environment, and a lower rate of productivity growth, i.e. “bad times,” increase the political support for more labor market rigidity only where labor appropriates of relatively large rents. Moreover, we demonstrate that when the status quo level of firing costs is relatively high, the preservation of a rigid labor market is favored by the employed with intermediate productivity, whereas all other workers favor more flexibility. The coming of better economic conditions need not favor the demise of high firing costs in rigid high-rents economies, because “good times” cut down the support for flexibility among the least productive employed workers. The model described provides some new insights on the comparative dynamics of labor market institutions in the U.S. and in Europe over the last few decades, shedding some new light both on the reasons for the original build-up of “Eurosclerosis,” and for its the persistence up to the present day.

**Keywords:** employment protection, firing costs, productivity, political economy, rents, volatility, growth, institutional divergence.

**JEL Classification:** D71, D72, E24, J41, J63, J65.

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# 1 Introduction

Employment protection legislation, or “firing costs,” varies considerably both across countries and over time. Within the OECD, stringent job security provisions are currently implemented in several Continental European countries, whereas other countries such as the U.K. and especially the U.S. have relatively flexible labor markets. There is also evidence that in Continental Europe firing costs have gradually become higher since the early 1970’s, the period traditionally associated with the build-up of “Eurosclerosis,” and have been reduced modestly in a number of countries since the beginning of the 1990’s.<sup>1</sup> In a few countries, such as France, there is no evidence that in recent years firing restrictions, and other forms labor market rigidities, have been reduced at all. The current persistence of the high firing costs legislated over past decades is particularly remarkable, since the issue of the reform of rigid labor market institutions toward more flexibility has been in recent times often at the top of the political agenda of many governments of Continental Europe. Yet, relatively little progress has been actually made in this direction, because of the decisive opposition of unions as well as of some pivotal political parties.<sup>2</sup>

In this paper, we present a general equilibrium model of dynamic labor demand, where the productivity of active firms varies over time according to a Geometric Brownian motion. The model, whose fundamentals and economic equilibrium are described and completely characterized in the first part of the paper, is then used to investigate how institutional and economic factors affect the *emergence* and the potential *persistence* of political support for employment protection regulations.

An essential feature of the model presented is that employed workers appropriate of a *rent*, i.e. of an economic benefit in excess of the utility of the unemployed. The rent appropriated by employed workers depends both on exogenous institutional factors, i.e. the “bargaining power” of labor, on the idiosyncratic productivity of the workers (who are ex-ante identical but become ex-post heterogeneous due to their variable labor market status), and on the level of the reservation productivity at which firms quit operating. The rent is also affected by the parameters of the stochastic process governing the evolution of the productivity of the firms, i.e. the drift and the instantaneous standard deviation, which characterize respectively the average rate of growth of productivity and its volatility, and describe the fundamentals of the economic environment.

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<sup>1</sup>See for example Cabellero and Hammour (1998), Blanchard (2000), and Blanchard and Wolfers (2000).

<sup>2</sup>See Saint-Paul (1996) for a discussion of the empirical evidence.

Job security provisions correspond to a legislated tax, the firing costs, imposed on the firms which decide to lay-off their employees. The level of firing costs is a key endogenous variable in the model and it is determined through a political process based on standard majority voting. Higher firing costs benefit the workers since they artificially extend the duration of jobs by reducing the reservation productivity, and therefore increase the total value of the rent appropriated by each employed worker. Yet, the decision of the workers to vote for a more rigid labor market, i.e. for a higher tax on firings, is a non-trivial one due to the general equilibrium effects of job security provisions. Higher firing costs increase the total cost of labor borne by firms, and therefore reduce job creation. As a result, in general equilibrium higher firing costs depress the exit rate from unemployment, to the detriment of both the utility of the unemployed and of the employed workers (whose prospects of future re-employment conditionally on being fired become less favorable).

The trade-off posed by higher firing costs is affected by the same variables determining the value of the rent appropriated by the employed workers. Of special importance, is the bargaining power of workers, i.e. their ability to extract rents from the firms employing them. Because the total value of the rent appropriated by the employed is proportional to their bargaining power, when this power is small enough there is little scope to protect jobs and the associated rents with legislated firing restrictions. In particular, we show that in this scenario the workers are unanimously in favor of zero firing costs, regardless on what their status quo level is. In the opposite case where the bargaining power of the workers is above a critical threshold, we show that workers split in two opposite coalitions, favoring respectively a rigid and a flexible labor market. In this scenario, individual preferences over firing costs depend on *labor market status*, i.e. whether a worker is employed or unemployed and, in the case of the employed workers, on their idiosyncratic productivity at the moment of voting, and on the *status quo level of firing costs*. Specifically, we show that when firing costs are relatively low to begin with, a transition to a rigid labor market is favored by the employed workers with idiosyncratic productivity below some threshold value. All the unemployed and the most productive employed are instead in favor of low firing costs. We also show that when the firing costs in place are high, a rigid labor market is preferred by the employed workers with *intermediate* productivity, i.e. belonging to a connected subset of the support of the distribution of productivity across active firms. Vice versa, a flexible labor market is preferred by an extreme coalition involving again all the unemployed, as well as the *more* and the *less* productive employed. Intuitively, regardless on what the status quo is, the unemployed

prefer to eliminate firing restrictions in order to induce firms to create more jobs, reducing the expected duration of their unemployment spell. The preferences of the employed are instead shaped by the assumption that the productivity of firms evolves over time according to a Geometric Brownian process. This assumption is important, since the (almost sure) continuity of paths of a Geometric Brownian process, implies that the expected duration of the job of an employed worker depends positively on his idiosyncratic productivity. As a result, the marginal gain of increasing firing costs for the employed workers decreases with their productivity. Because highly productive workers gain relatively little from higher rigidity, they prefer a more flexible labor market. The employed workers with productivity below some threshold create the potential coalition for rigidity, since higher firing costs may increase their total utility by providing them valuable insulation from the risk of job loss due to some future bad shocks. However, if the economy is relatively rigid to begin with, the *least productive* employed workers are also in favor of a flexible labor market. The reason is that, since their job is about to end anyway, they prefer to make the labor market more flexible to the rigid status quo, in order to improve their future re-employment prospects.

The analysis of the politico-economic equilibrium reveals some additional interesting and surprising insights. In particular, we show that a *complementarity* arises in the equilibrium of the model between the volatility of productivity and labor market flexibility, in the sense that in a more turbulent environment, the positive effect that more flexibility has on job creation is magnified, to the benefit of all workers. A similar complementary, though, arises since higher volatility also magnifies the rent erosion effect caused by a reduction of firing costs. Where the power of rent extraction of the employed is relatively high, the magnification of the marginal loss from flexibility due to rent erosion, dominates over the magnification of the marginal gain due to job creation. As a result, labor market flexibility enjoys *less* political support overall, precisely because rents are relatively important for the workers. The opposite is true where the employed are able to extract relatively low rents from firms, in which case the magnification of the marginal gain from flexibility dominates over the magnification of the marginal loss, making labor market flexibility more appealing. In addition, we also show that a *substitutability* arises in the equilibrium of the model between productivity growth and labor market flexibility, in the sense that higher productivity growth reduces both the positive and the negative marginal effect of flexibility on the welfare of the employed. Again, if the rent extraction power of the workers is large enough, the marginal loss caused by flexibility due to rent erosion, decreases relative to the marginal gain due to job creation, increasing the political support for a *more*

flexible labor market, and vice versa.

The broader significance of this result is that how economic shocks, i.e. unexpected changes in the parameters governing the evolution of the productivity of the firms, affect the political equilibrium of the model, depends on the power of labor to extract rents. This finding can help to explain the divergent evolution of the labor market institutions in high and low rents economies, i.e. Continental Europe vs. the U.S. and the U.K., since the early 1970's, in response to similar negative aggregate shocks generating high volatility and low growth.

Finally, we show that, whereas bad business conditions may favor the demise of relatively flexible labor market institutions, the opposite type of shock need not help to create more political support for a reversion to labor market flexibility in a rigid economy. This is so since good economic conditions, i.e. low volatility and fast productivity growth, cut down the political support for low firing costs among the least productive employed workers. Intuitively, this is because these workers expect to earn more *future* rents conditionally on remaining employed and, as a result, prefer to keep in place the high barriers protecting their relatively fragile *present* employment status.

This paper is related to a variety of different contributions including primarily previous models of political economy of labor market institutions, such as Lindbeck and Snower (1988) and Saint-Paul (1993),<sup>3</sup> as well as the more recent contributions of Saint-Paul (1999 and 2002). The main difference between my paper and Saint-Paul (1999 and 2002), is that his model addresses the question of how the preferences for employment protection are affected by the rate of growth of *embodied* productivity within a vintage capital model, rather than by idiosyncratic productivity fluctuations as in my model. By focusing on a different form of productivity growth, *disembodied* in firms rather than embodied, I obtain a number of different comparative statics results. In particular, whereas Saint-Paul finds that higher productivity growth reduces unambiguously the political support for employment protection regulation, I find that how growth affects the political equilibrium generally depends both on the bargaining power of labor, and on the status quo level of firing costs. In addition, Saint-Paul does not investigate how volatility, which plays an essential role in my model, affects the politico-economic equilibrium. Other papers such as Hassler and Rodríguez Mora (1999), and Hassler, Storesletten, Rodríguez Mora and Zilibotti (2005) are also related, but they both focus on the political economy of unemployment insurance rather than of firing costs as we do here.

Secondly, the paper is related to the important models of dynamic labor demand of Bentolila

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<sup>3</sup>See also Saint-Paul (2000) for a survey of this literature.

and Bertola (1990) and of Mortensen and Pissarides (1994). The paper of Bentolila and Bertola (1990) presents a partial equilibrium model with a stochastic structure identical to the one assumed here, which I extend to allow for the endogenous determination of wages, labor market flows and firing costs. Mortensen and Pissarides' (1994) model has a dynamic general equilibrium structure, based on the assumption that firms experience productivity shocks described by a homogenous Poisson process, rather than by a Geometric Brownian process as assumed here. This assumption implies that the expected duration of a match does not depend on the level of its idiosyncratic productivity, unlike in my model where more productive workers expect to remain employed longer. Because in a Mortensen and Pissarides' type of setup all employed workers face the same exposure to productivity shocks, they all have potentially the same preferences over employment protection legislation. Conversely, as already explained, in my model more productive workers tend to demand less firing costs due to their relative insulation from the risk of job loss. In addition, some results relative to the comparative statics around the economic equilibrium are different. For example, in my model higher volatility has a *positive* effect on job creation overall, whereas in Mortensen and Pissarides' it has the opposite effect.<sup>4</sup>

Also related in the macro-labor literature are the papers of Ljungqvist and Sargent (1998), of Hopenhayn and Rogerson (1993), of Bertola (1994) and of MacLeod, Malcomson, and Gomme (1994). Ljungqvist and Sargent (1998) argue the surge of unemployment in Europe since the 1970's can be explained with how layoff taxes and unemployment compensation linked to past earnings interact with an increase in economic turbulence. Hopenhayn and Rogerson (1993) address the questions of how taxes on job destruction affect social welfare in a dynamic general equilibrium model of labor demand. Bertola (1994) investigates the efficiency costs and distributional effects of obstacles to labor mobility, in a model of endogenous growth with diversifiable microeconomic uncertainty. MacLeod, Malcomson, and Gomme (1994) investigate how changes in the economic environment affect wages and employment in efficiency wage models.

Lastly, the paper is related to the political economy literature on inefficient redistribution (e.g. Coate and Morris, 1995; Acemoglu and Robinson, 2001), and on the persistence of policies and institutions (e.g. Fernandez and Rodrik, 1991; Coate and Morris, 1999; Bénabou, 2000; Acemoglu, Ticchi and Vindigni, 2006; Bruegemann, 2007). In particular, in an influential paper Bénabou (2000) demonstrates that “unequal societies,” featuring very different degrees

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<sup>4</sup>See also the comprehensive discussion of search and matching models of the labor market presented and their properties in Pissarides (2000).

of fiscal redistribution of income and of inequality, can arise and persist in a dynamic model of political economy of taxation, depending on the initial degree of income inequality. Moreover, in a recent paper, Bruegemann (2007) also addresses the specific question of the persistence of rigid labor market institutions, under alternative assumptions regarding the wage setting rule.

The paper is organized as follows. Section 2 lays down the foundations of the model, whose economic equilibrium is obtained and characterized in section 3. The political equilibrium, its properties and some important applications of the model are characterized in sections 4, 5 and 6 respectively. Section 7 concludes. All the proofs are either in the main text or are reported in the appendix.

## 2 The Economy

### 2.1 Basic Environment

The economy is a small and open one, populated by a continuum of measure one of risk neutral workers who always consume all of their disposable income. Workers can be employed or unemployed, and discount future welfare at rate  $r$  equal to the real interest rate. Hence, letting  $\{y_u\}_{u=t}^{\infty}$  denote the uncertain future income stream of a worker, his preferences can be represented as

$$\mathbb{E}_t \left\{ \int_t^{\infty} e^{-r(u-t)} y_u du \right\}, \quad (1)$$

where  $\mathbb{E}_t$  denotes the expected value operator, conditional on the information available at  $t$ . Firms are created by a small set of risk neutral entrepreneurs, by paying a fixed setup normalized at zero. The available production technology is Leontief, allowing a firm to produce some amount of output per unit of time by hiring one worker only. There are no search frictions, are therefore firms fill up their vacancy instantaneously. The productivity  $x$  of each firm is normalized to one at moment when the firm is created, but it varies over time due to the realization of random idiosyncratic shocks. Specifically,  $x$  follows a Geometric Brownian process, whose stochastic differential is represented by

$$dx = \mu x dt + \sigma x dW, \quad (2)$$

where  $W$  stands for a Wiener process. The parameters  $\mu \in \mathbb{R}_+$  and  $\sigma \in \mathbb{R}_{++}$  indicate respectively the drift and the instantaneous standard deviation of  $x$ . To ensure the existence of an equilibrium, we restrict the parameters in question, by assuming that  $\mu < \sigma^2/2$ .



Because productivity is variable, a firm may eventually decide to stop producing and to lay-off the worker. When this event happens, the firm pays the mandatory firing costs  $F \geq 0$  for dismissing the worker, which represent a pure deadweight loss, i.e. the corresponding income is entirely wasted. The firing costs  $F$  are chosen by the society through a standard political process based on majority voting, described in greater detail in Section 4.

The value of a firm  $\mathcal{J}(\cdot)$  active at time  $t \in \mathbb{R}_+$ , i.e. the expected present discounted value of the stream of profits gross of the layoff cost, as a function of its productivity  $x = x_t$ , can be written as

$$\mathcal{J}(x) = \sup_{\bar{T} \in [t, \infty)} \mathbb{E}_t \left\{ \int_t^{\bar{T}} e^{-r(u-t)} [x - w(x)] du - e^{-r(\bar{T}-t)} F \right\}, \quad (3)$$

where the supremum is taken over the set of possible stopping times  $\bar{T}$ , at which the firm can decide to quit producing and to lay-off the worker, i.e.  $[t, \infty)$ . By standard arguments,<sup>5</sup> the value function  $\mathcal{J}(\cdot)$  satisfies the following Bellman-Wald functional equation

$$r\mathcal{J}(x) = \max \left\{ x - w(x) + \frac{1}{dt} \mathbb{E}(d\mathcal{J}), -rF \right\}, \quad (4)$$

which characterizes the optimal stopping problem of the firm. The right-hand-side of (4) is the maximum between the continuation value of the asset corresponding to the value of firm, and the flow-equivalent (or annuity value) of the firing costs  $F$ . The continuation value is equal to the flow payoff generated by the asset, plus the expected capital gain. The solution of the optimization problem represented by equation (4) involves implementing a barrier-control policy. The firm closes down, laying-off its worker and paying the mandatory firing costs  $F$  if, and as soon as, its productivity reaches a reservation level  $R$ , corresponding to an optimally set threshold. The optimal stopping rule of the firm is characterized in the appendix of the paper, where we solve the free-boundary problem represented by the differential equation associated with the functional equation (4), and the related optimality conditions (i.e. the “value matching” and the “smooth pasting” conditions). For future reference, we define the random calendar time  $\bar{T}_x(R)$  at which the level of stochastic process describing the productivity of a firm active at time  $t \in \mathbb{R}_+$  with  $x_t = x$ , is absorbed by the barrier  $R$  as

$$\bar{T}_x(R) \equiv \inf \{u \in [t, \infty) : x_u = R | x_t = x\}. \quad (5)$$

The value of a firm  $\mathcal{J}(\cdot)$  also satisfies the initial value condition following from the standard assumption of free entry, which implies that firms earn no pure profits in equilibrium, since

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<sup>5</sup>See for example Dixit (1993), Dixit and Pindyck (1994), Peskir and Shiryaev (2006).

the ex-ante value of job creation, i.e. corresponding to the initial level of productivity  $x = 1$  is equalized to the zero setup cost. Formally, free entry of vacancies implies that

$$\mathcal{J}(1) = 0. \quad (6)$$

## 2.2 Wage Setting Mechanism

We now describe the wage setting mechanism. It is useful to begin by breaking down expression (1) into a pair of recursive equations satisfied by the values of employment and of unemployment. The value  $\mathcal{W}(x)$  of working in a firm with idiosyncratic productivity  $x \in (R, \infty)$ , and the value  $\mathcal{U}$  of unemployment satisfy the following system of functional equations

$$r\mathcal{W}(x) = w(x) + \frac{1}{dt}\mathbb{E}(d\mathcal{W}), \quad (7)$$

and

$$r\mathcal{U} = b + \theta(\mathcal{W}(1) - \mathcal{U}), \quad (8)$$

where  $w(x)$  is the wage rate paid by the firm to the worker,  $b$  is the exogenous level of unemployment compensation (or value of leisure), and  $\theta$  stands for the exit rate out of unemployment, which is endogenous to the model.

I assume the same wage setting mechanism hypothesized in Saint-Paul (1999), where it is assumed that a worker employed in a firm with productivity  $x$  earns a salary such that the corresponding value of employment is equal to the value of unemployment plus a fraction  $\beta$  of the expected present discounted value of the output stream produced by the firm. The sharing rule in question, represented by equation (9) below, can be given a micro-foundation as the expression of the rents that firms need to pay to the workers to cope with an underlying *moral hazard* problem, as in the spirit of efficiency wage models (e.g. Shapiro and Stiglitz, 1984).<sup>6</sup> The sharing rule (9) has the important implication that firing costs affect wages only indirectly, i.e. by reducing the reservation productivity, rather than also directly, i.e. by affecting the relative bargaining power of workers and firms.<sup>7</sup> While the effect of employment protection

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<sup>6</sup>As equation (9) makes transparent, the sharing rule that we assume can be interpreted as a generalization of the standard sharing rule applying in efficiency wage models à la Shapiro and Stiglitz (1984). The main difference is that whereas in a standard efficiency wage model the employed obtain a rent equal to a *fixed* markup over the value of unemployment, (9) implies that the rent is variable, reflecting the idiosyncratic productivity of the firm. See Saint-Paul (1999) for the exposition of the micro-foundation leading to the sharing rule (9).

<sup>7</sup>This is the case, for example, if wages are set with Nash bargaining, in which case higher firing costs (which firms are supposed to pay), make workers stronger at the bargaining table by increasing the cost of a negotiation breakdown for the firms.

legislation over the bargaining power of the firms is also potentially interesting, I intend to focus the attention only on the role of firing costs in extending the duration of jobs, which the sharing rule (9) allows me to do.

More formally, I assume that the wage  $w(x)$  paid to a worker by a firm with productivity  $x$  is such that

$$\mathcal{W}(x) = \mathcal{U} + \beta V(x). \quad (9)$$

In this expression,  $\mathcal{W}(x)$  and  $\mathcal{U}$  are defined recursively by (7) and (8), and

$$V(x) = \mathbb{E}_t \left\{ \int_t^{\bar{T}_x(R)} e^{-r(u-t)} x_u du \mid x_t = x \right\},$$

represents the expected present discounted value of the future output stream generated by a firm having at time  $t$  a productivity level  $x_t = x$ , up to the absorption time  $\bar{T}_x(R)$  defined by (5), computed with respect to the probability distribution of  $\bar{T}_x(R)$ . The parameter  $\beta \in (0, 1)$  in (9) represents the power of rent extraction of employed workers.

An immediate consequence of the moral hazard problem leading to the sharing rule (9) is the existence of involuntary unemployment: the unemployed are willing to work for a wage lower than the wage paid to the employed, but firms are nonetheless unwilling to hire them. Notice that, because at the moment when the absorbing barrier  $R$  is reached, the worker is fired and the match broken, it is the case that  $V(R) = 0$ . This fact and the sharing rule (9) imply that the following terminal condition also holds

$$\mathcal{W}(R) = \mathcal{U}, \quad (10)$$

according to which the value of employment at the reservation productivity  $R$  is equal to the value of unemployment. As demonstrated in the appendix, the wage schedule implied by the sharing rule (9) reads

$$w(x) = b + \theta \beta V(1) + \beta x. \quad (11)$$

Closed-form expressions are available both for  $V(\cdot)$  and for  $\mathcal{J}(\cdot)$  and read, respectively

$$V(x) = \frac{x}{r - \mu} - \frac{R^{1-\alpha} x^\alpha}{r - \mu}, \quad (12)$$

and

$$\mathcal{J}(x) = \frac{(1 - \beta)x}{r - \mu} - \frac{b}{r} - \frac{\theta \beta}{r} \left( \frac{1 - R^{1-\alpha}}{r - \mu} \right) - \frac{(1 - \beta)R^{1-\alpha} x^\alpha}{\alpha(r - \mu)}, \quad (13)$$

where  $\alpha$  corresponds to the negative root of the characteristic polynomial associated with the differential equation satisfied by  $\mathcal{J}(\cdot)$ .<sup>8</sup>

### 3 Economic Equilibrium

#### 3.1 Aggregation

In this subsection, we begin the description of the economic equilibrium of the model, assuming that a steady state featuring positive job creation and job destruction exists. In our model economy each firm is created at some point in time, and experiences thereafter the realization of idiosyncratic shocks to its productivity, until the time when absorbing barrier  $R$  is reached. While the duration of the life-span of each firm is random, the evolution over time of the cohort of firms created at the same point in time is deterministic, since every cohort of new firms is formed by a continuum of units. Therefore, by a law of large numbers, the deterministic fraction of the firms of each cohort that are still active at any point in time following their creation, corresponds to the survival probability of a firm from the same cohort up to that time.

Because the transition density function of the stochastic process (2) describing the dynamics of productivity is time-homogenous, the random time  $T(R) \equiv \bar{T}_1(R) - t$  elapsed *since the time  $t$  of creation* of a firm (when productivity is  $x_t = 1$ ) at which absorption takes place, does not depend on the calendar time of creation of the firm. Therefore, we can write the probability distribution of  $T(R)$  as follows

$$\Pr\{T(R) > u\} = \int_R^\infty p(1, \xi; u) d\xi,$$

where  $p(1, \cdot; u)$  denotes the transition density function of  $x$ , conditional on the absence of absorption since the moment of creation of the firm  $t$  up to time  $t + u$ .

At time  $s$ , the flow of workers from unemployment into employment, equivalent to the mass of newly created production units, has measure  $\theta_s(1 - L_s)$ , where  $L_s$  denotes the total mass of employed workers at  $s$ . Therefore, assuming that the economy begins operating at time 0, the total employment  $L_t$  at time  $t$  can be decomposed as the (integral) sum of the firms created over the period  $[0, t]$ , weighting the mass of firms of each cohort<sup>9</sup> by the survival probability up to time  $t$  of their “representative” unit, so that

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<sup>8</sup>The expression of  $\alpha$  is reported in equation (43) in the appendix.

<sup>9</sup>We remind that each active firm hires one worker only.

$$L_t = \int_0^t \theta_{t-s} (1 - L_{t-s}) \Pr \{T(R) > s\} ds. \quad (14)$$

In the steady state, all aggregate labor market outcomes are stationary, and therefore  $L_t = L$ ,  $\theta_t = \theta$ , and  $\delta_t = \delta$ , where  $\delta_t$  indicates the aggregate job destruction rate. Moreover, the labor market flows-balance condition

$$\delta L = \theta (1 - L), \quad (15)$$

equating the number of jobs destroyed per unit of time,  $\delta L$ , to the number of jobs created, also applies in the steady state. Combining the steady state form of expression (14), obtained by imposing stationarity and letting  $t \uparrow \infty$ , and equation (15), the aggregate steady state job destruction rate  $\delta$  can be written as

$$\delta = \frac{1}{\int_0^\infty \Pr \{T(R) > t\} dt}. \quad (16)$$

### 3.2 Characterization

Given the level of firing costs implemented (which will be determined later in the political equilibrium), the economic equilibrium of the model is defined by a pair of equations in two endogenous variables, the reservation productivity  $R$  and the exit rate from unemployment  $\theta$ . The first of these equations is the free entry condition, which can be computed from (6) and (13) and reads

$$\frac{(1 - \beta)}{r - \mu} - \frac{b}{r} - \frac{\theta \beta}{r} \left( \frac{1 - R^{1-\alpha}}{r - \mu} \right) - \frac{(1 - \beta) R^{1-\alpha}}{\alpha (r - \mu)} = 0. \quad (17)$$

The second equation corresponds to the “value matching” condition, which arises from the solution of the optimal stopping problem of the firm (see the appendix), and establishes the continuity of the firm’s value function upon closing down. This equation reads

$$\frac{(1 - \beta) R}{r - \mu} - \frac{b}{r} - \frac{\theta \beta}{r} \left( \frac{1 - R^{1-\alpha}}{r - \mu} \right) - \frac{(1 - \beta) R}{\alpha (r - \mu)} = -F. \quad (18)$$

Equation (17) can be used to obtain the expression of the rate of job creation, as a function of the reservation productivity, or

$$\theta = \frac{r (1 - \beta) R^{1-\alpha} + \alpha ((r - \mu) b - r (1 - \beta))}{\alpha^+ \beta (1 - R^{1-\alpha})}, \quad (19)$$

where  $\alpha^+ \equiv |\alpha|$ . Equations (18) and (19) can then be combined to obtain a single equation defining implicitly the equilibrium value of  $R$ , and reading

$$\frac{1-\beta}{r-\mu} \left( 1 - R + \frac{R - R^{1-\alpha}}{\alpha} \right) = F. \quad (20)$$

Equation (20) states that the expected present discounted value of the flow of gross profits of a firm is equal to the firing costs (the only cost borne by firms other than wages). The economic equilibrium of the model has a recursive structure. Equation (20) defines a downward-sloping relation between  $R$  and  $F$ , which determines the unique equilibrium value of the reservation productivity, as a function of a set of exogenous parameters including firing costs (which are determined endogenously in the political equilibrium, but are still treated as given here). Finally, the equilibrium value of  $\theta$  can be computed using the equilibrium value of  $R$  and equation (18), which defines a strictly upward sloping locus in the  $(R, \theta)$  plane.

**Remark 1** Since the productivity of a firm is always non-negative,<sup>10</sup> the reservation productivity  $R$  has a lower bound at zero, and equation (20) implies the level of firing costs corresponding to  $R = 0$  reads

$$\tilde{F} = \frac{1-\beta}{r-\mu}. \quad (21)$$

If  $F = \tilde{F}$ , firms never close down and, as a result, the model has a steady state where the rate of job destruction is zero and all workers are employed, contrary to the assumption made that a steady state exists with strictly positive job creation and destruction. To ensure the existence of a stationary economic equilibrium with the desired properties, the following restriction is imposed on  $F$ .

**Assumption 1**  $F \leq \hat{F}$ , where  $\hat{F} \in (0, \tilde{F})$ .

We conclude this subsection by reporting the closed-form expression of  $\delta$ , which is computed in the appendix, along with the closed-form expression of the ergodic probability density function of productivity across active firms, which is also computed in the appendix of the paper, when  $R < 1$ , i.e. firing costs are strictly positive.

**Proposition 1** *If  $R < 1$ , the steady state aggregate job destruction rate,  $\delta$ , reads*

$$\delta = \frac{1}{\hat{R}^+} \left( \frac{\sigma^2}{2} - \mu \right), \quad (22)$$

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<sup>10</sup>This follows immediately from the assumptions made relative to the initial value and to the dynamics of  $x$ .

where  $\hat{R}^+ \equiv |\ln R|$ , and the ergodic cross-sectional distribution of productivity across firms,  $\Psi(\cdot)$ , has probability density function  $\psi(\cdot)$  represented by<sup>11</sup>

$$\psi(x) = \frac{1}{\hat{R}^+} \frac{1 \cdot \mathcal{I}_{\{R \leq x \leq 1\}} + x^{\frac{2\eta}{\sigma^2}} \cdot \mathcal{I}_{\{1 \leq x\}} - x^{\frac{2\eta}{\sigma^2}} e^{-\frac{2\eta}{\sigma^2} \hat{R}^+}}{x}, \quad (23)$$

where  $\eta \equiv [\mu - (\sigma^2/2)]$ , and  $\mathcal{I}$  denotes the indicator function defined in the standard way.

**Proof.** See appendix. ■

**Remark 2** Equation (20) implies that  $R = 1$  in the limit case where  $F = 0$ , i.e. the reservation productivity is equal to the standardized initial productivity level. As a result, both the rate of job creation (19) and the rate of job destruction (22) are infinite, and therefore the expected duration of any spell of employment and of unemployment is zero. Moreover, the cross-sectional distribution of productivity across employment has a mass-point at  $x = 1$ .

### 3.3 Comparative Statics

The economic equilibrium of the model has a number of comparative statics properties, some of which are relatively non-standard. These properties are discussed next, under the qualification that the firing costs  $F$  are always held constant as the parameters of interests are allowed to vary around the economic equilibrium. Also, we focus the attention on the generic case where  $F > 0$ , leaving aside the limit case described above of  $F = 0$ , where labor market flows are both infinite.

1. Higher firing costs  $F$  reduce the reservation productivity  $R$  since firms prefer to hold on longer when layoffs are more costly; as a result, both the aggregate rate of job destruction rate  $\delta$  and the exit rate from unemployment  $\theta$  (which are increasing in  $R$ ) fall. It follows from equation (15) that higher firing costs have overall ambiguous effects on the level of equilibrium employment.<sup>12</sup>
2. A higher value of the rent extraction power  $\beta$  reduces the reservation productivity, the job destruction rate and the exit rate from unemployment. The reservation productivity falls since firms prefer to hold on for a longer time, as they need to compensate for the resulting loss of flow-profits due to a higher  $\beta$ . The job destruction rate falls as a result

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<sup>11</sup>I am especially grateful to Bjoern Bruegemann and Simone Scotti for help in the computation of the ergodic distribution of productivity.

<sup>12</sup>This is a very well known and general result, first pointed out by Bentolila and Bertola (1990) and more recently, among others, by Blanchard and Portugal (2001).

of the fact that the reservation productivity is lower. The exit rate from unemployment decreases with  $\beta$  both because of the direct negative effect of  $\beta$  on profitability, and because of the general equilibrium effect that  $\beta$  has on  $\theta$  going through the reservation productivity, which is positively related with  $\theta$  by (19).

3. Higher volatility  $\sigma$  *decreases* the equilibrium reservation productivity, because in a more turbulent environment the option value of a job is higher for the firm. It follows that the impact of  $\sigma$  on the steady state aggregate rate of job destruction  $\delta$  is ambiguous. This is because  $\delta$  increases with  $\sigma$  in partial equilibrium, i.e. holding  $R$  constant, but it also decreases with  $\sigma$  through  $R$ , due to the negative effect that volatility has on the reservation productivity, and to the fact that  $\delta$  increases with  $R$ .
4. Higher volatility *stimulates* job creation, i.e. it raises  $\theta$ . This is a consequence of the *convexity effect* of volatility, which increases the value of a firm and therefore drives up job creation.<sup>13</sup> Notice that this is a general equilibrium effect, which dominates over the negative partial equilibrium effect that  $\sigma$  has on  $\theta$  due to the fact that  $\theta$  increases with  $R$ , and that  $R$  decreases in equilibrium with  $\sigma$  as we already know.
5. A higher value of the drift coefficient  $\mu$  increases the equilibrium reservation productivity since higher productivity growth raises profitability, which means that  $R$  must increase in order to restore the economic equilibrium. The impact of  $\mu$  on  $\delta$  is instead ambiguous, since  $\mu$  has a negative direct effect on it, but also a positive indirect effect due to the increment of  $R$ , which leads to more job destruction. The impact of  $\mu$  on  $\theta$  is also ambiguous, since both the value of output and the cost of labor are increasing in  $\mu$ .

Notice that the set of comparative statics results implies that the effects of all the parameters considered, not just of firing costs, on equilibrium employment are *a priori* ambiguous.

## 4 Politics

### 4.1 The Political Mechanism

We assume that a given level of firing costs  $F = F_0$  is initially implemented, representing the status quo level of employment protection. The status quo value of  $F$  may be changed as a result of a majority voting process. We assume that voting on firing costs takes place only

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<sup>13</sup> Conversely, in a matching model with endogenous separations, and where idiosyncratic uncertainty is described by a homogenous Poisson process, a higher arrival rate of productivity shocks reduces job creation (see for example Pissarides, 2000).



once, immediately after an *unexpected* shock to the exogenous variables of the model occurs (in particular, the rent extraction power of the workers, and the drift and standard deviation of the Brownian process describing the evolution of productivity), when the economy is in the economic equilibrium corresponding to  $F = F_0$ .<sup>14</sup> The new legislated firing costs correspond to any point of the policy space, i.e. the interval  $[0, \hat{F}]$ . The assumption that voting takes place immediately after a shock hits the economy reflects the fact that it is optimal for the majority of workers to vote immediately rather than to wait. This is because, if a majority is in favor of changing the status quo, it is strictly better-off by doing it as soon as possible; vice versa, if the majority is in favor of preserving the status quo, it gains nothing by voting later on.

For analytical reasons, it is convenient to assume that workers vote for the level of the *effective* level of employment protection, i.e. the reservation productivity  $R$ , rather than for legal employment protection, i.e. the level of firing costs  $F$ . Since a one-to-one relation between  $F$  and  $R$  exists according to equation (20), voting on  $F$  is equivalent to voting on the corresponding level of  $R$ . The relevant policy space is thus the interval  $[\underline{R}, 1]$ , where  $\underline{R}$  is defined as the reservation productivity corresponding to the maximum feasible level of firing costs  $\hat{F}$ .

It must be emphasized at this point that if the legislated firing costs differ from  $F_0$ , the transition to the new politico-economic equilibrium is instantaneous for  $\theta$ ,  $\mathcal{J}(x)$ ,  $w(x)$ ,  $\mathcal{W}(x)$  and  $\mathcal{U}$ , which are all functions of jump-variables only.<sup>15</sup> Since the welfare of all the workers jumps instantaneously to the steady state level corresponding to any level of firing costs different from  $F_0$ , in deciding how to vote workers simply compare their utility across different steady states. In particular, the transitional dynamics to the new equilibrium of all the state-variables, i.e. the cross-sectional distribution of productivity, and the level of employment (which converge gradually to the new ergodic distribution and to the new steady state respectively), does not affect the voting decision of any worker.

It is not possible to characterize the political equilibrium of the model using the median voter theorem since, as we already know, the preferences of a set of positive measure of agents

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<sup>14</sup>The assumption that voting takes place only once rules out the interesting but potentially complicated effects that the anticipation of the future political equilibria has on the current voting decision of the workers (see Hassler, Storesletten, Rodríguez Mora and Zilibotti, 2000, and Acemoglu, Ticchi and Vindigni, 2006, for examples of dynamic political games based on repeated majority voting).

<sup>15</sup>Equation (20) implies that  $R$  depends only on  $F$  and therefore it immediately adjusts to the steady state value corresponding to the new value of  $F$ . Equation (19) implies that  $\theta$  only depends on  $R$  and therefore it also adjusts instantaneously. Finally, the expressions of the value functions of all the workers show that the only endogenous variables on which they depend are  $R$  and  $\theta$ .

do not satisfy the single-peakness (and neither the single-crossing) property. Nonetheless, we are able to demonstrate that the social preferences over employment protection regulation induced by majority voting do not indeed cycle, i.e. that a political equilibrium always exists. In particular, it is possible to demonstrate that a unique Condorcet winner supported at *unanimity* exists *regardless* on what the status quo level of firing costs is, provided the rent extraction power of the workers is below some threshold value. When the rent extraction power of the workers exceeds the threshold value in question, a unique political equilibrium still exists, but the corresponding policy is not voted at unanimity and, more importantly, the equilibrium may depend on the status quo level of employment protection. To make progress in the characterization of the political equilibrium, we compute next the closed-form expressions of the value functions of all workers.

## 4.2 The Structure of the Preferences over Labor Market Regulation

In this subsection, we describe the preferences over employment protection regulations of all workers. We begin by computing the values of the unemployed and of the employed workers. Combining equations (7), (8), (9), (11) and eliminating  $\theta$  in the resulting expression by using (19), the value of unemployment  $\mathcal{U}$  can be written as

$$\mathcal{U} = \frac{1 - \beta}{r - \mu} - \frac{(1 - \beta) R^{1-\alpha}}{(r - \mu) \alpha}. \quad (24)$$

Straightforward differentiation of (24) shows that the value of unemployment is strictly increasing in  $R$ . Intuitively, unemployed workers would be strictly better-off in a fully flexible labor market where layoffs are not constrained in any way and where therefore the exit rate from unemployment is as high as it can be.

The value of employment  $\mathcal{W}(\cdot)$  is instead computed by combining equations (12) and (24), and reads

$$\mathcal{W}(x) = \frac{1 - \beta}{r - \mu} - \frac{(1 - \beta) R^{1-\alpha}}{(r - \mu) \alpha} + \beta \frac{x - R^{1-\alpha} x^\alpha}{r - \mu}. \quad (25)$$

In the following, we will occasionally make the dependence of the value of the employed on  $R$  explicit, by writing  $\mathcal{W}(x)$  as  $\mathcal{W}(x|R)$ , to denote the value of employment in a firm with productivity  $x$ , conditionally on  $R$ . Similarly, we will sometime use the expression  $\mathcal{U}(R)$  to denote the value of unemployment, also conditionally on  $R$ .

Equation (25) allows us to determine how the welfare of employed workers depends on a *marginal* increment in  $R$ .

**Lemma 1** *Let  $R = R_0$  denote the status quo reservation productivity. All workers employed in firms with productivity  $x \in (x^*, \infty)$ , where*

$$x^* = [(1 - \beta) / (\beta \alpha^+)]^{\frac{1}{\alpha}}, \quad (26)$$

*benefit strictly from a marginal increment in labor market flexibility (i.e. an infinitesimally higher value of  $R$ ), all workers in firms with productivity  $x \in (R_0, x^*)$  are made strictly worse-off, and all workers in firms with productivity  $x = x^*$  are indifferent.<sup>16</sup>*

**Proof.** A straightforward differentiation of equation (25) shows that  $\partial \mathcal{W}(x|R) / \partial R \geq 0$  for any  $R$  if  $x \geq x^*$ , and that  $\partial \mathcal{W}(x|R) / \partial R = 0$  if  $x = x^*$ . ■

Lemma 1 tells us that the workers employed by relatively productive firms (i.e. with  $x > x^*$ ) are made better-off if the labor market becomes *marginally* more flexible, while the workers employed by relatively unproductive firms are made worse-off. Intuitively, employment protection involves benefits, due to the extension of the duration of the rent appropriated by the employed, but also costs, due to its adverse general equilibrium effect on job creation. By differentiating equation (25), the total effect of a marginal increment of  $R$  on the welfare of the workers with productivity  $x$  can be decomposed into two parts, corresponding respectively to the marginal gain,  $\partial \mathcal{U}(R) / \partial R$ , and to the marginal loss, proportional to  $\partial V(x|R) / \partial R$ . The gain of increasing  $R$  is *the same* for all workers, independently from their individual productivity, since it is due entirely to the corresponding variation of the value of unemployment (expressed by the sum of the first two terms in (25)), which as we know is positive. Conversely, the loss caused by more flexibility, due to the reduction of the value of the rent appropriated by the employed (i.e. the fourth term in (25)), can be shown to be *decreasing* in  $x$ .<sup>17</sup> This implies that relatively more productive workers lose relatively *less* by a relaxation of the firing discipline, and explains why the workers with productivity above the threshold  $x^*$  are better-off with more flexibility, and vice versa.<sup>18</sup>

From what has been said so far, it may appear that all workers with productivity below  $x^*$  are always harmed by an increment of  $R$ . However, this is not the case because Lemma 1 only determines how a *marginal* increase in labor market flexibility affects the welfare of employed

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<sup>16</sup>If  $R_0 \geq x^*$ , the interval  $(R_0, x^*)$  is of course empty, in which case all employed workers are made strictly better-off if firing costs are relaxed infinitesimally.

<sup>17</sup>This follows from a straightforward differentiation of  $\partial V(x|R) / \partial R$  with respect to  $x$ .

<sup>18</sup>This result depends on the nature of the stochastic process governing the dynamics of productivity, and in particular on the persistence proper of geometric Brownian motion. If the realizations of  $x$  were governed by a homogeneous Poisson process, then an infinitesimal increment of  $R$  would have the same effect on the lifetime utility of *all* of the employed.

workers, as opposed to a *discrete* increase in  $R$ . If  $R$  increases from  $R = R_0$  to a higher value  $R = R'$ , i.e. if labor market flexibility increases by a non-infinitesimal amount, then a set of jobs of non-zero measure, corresponding to the firms with productivity in the interval  $[R_0, R']$ , that were initially prevented from closing down by the tighter firing restrictions, is instantaneously destroyed. It can be shown that a set of positive measure of least productive workers exists, who are better-off if  $R = R'$  than they are in the status quo, despite the fact that they are fired if the reform is implemented. Intuitively, thus is because the function  $\mathcal{W} = \mathcal{W}(x|R_0)$  is continuous in  $x$  over the range  $[R_0, \infty)$ .<sup>19</sup> This means that the welfare of the employed workers with productivity in a left-neighborhood of  $R_0$  of small radius is approximately equal to  $\mathcal{U}(R_0)$ . Moreover, voting involves here the choice between two alternatives which, for the workers with productivity  $x \simeq R_0$  is approximately equivalent to the choice between being *unemployed* in a relatively rigid (i.e. with  $R = R_0$ ) and in a relatively flexible economy (i.e. with  $R = R'$ ). Given that the value of the unemployed is everywhere increasing in  $R$ , it is clear that the workers employed by firms whose idiosyncratic productivity is sufficiently close to the status quo reservation productivity  $R_0$  will vote for a *more* flexible labor market.

This argument can be stated more formally by considering the expression of the productivity level  $x'_0$  at which workers indifferent between the two policy alternatives considered above. Letting  $\mathcal{W}(\cdot|R_0)$  denote the value of employment in the status quo labor market, as a function of idiosyncratic productivity, and letting  $\mathcal{U}(R')$  denote the value of unemployment in the reformed labor market, the threshold  $x'_0$  is defined implicitly by the equation  $\mathcal{W}(x'_0|R_0) = \mathcal{U}(R')$ , which can be written as

$$x'_0 - (R_0)^{1-\alpha} (x'_0)^\alpha = (x^*)^\alpha \left[ (R')^{1-\alpha} - (R_0)^{1-\alpha} \right]. \quad (27)$$

Since the left-hand-side of this equation is strictly increasing in  $x'_0$ , and equal to zero if  $x'_0 = R_0$ , whereas its left-hand-side is strictly positive, equation (27) has always a unique solution over the range  $(R_0, \infty)$ . Hence, it exists a semi-closed set of positive measure  $[R_0, x'_0)$ , such that the workers employed in firms with productivity in this interval are strictly better-off as unemployed in the more flexible labor market with  $R = R'$ , than as employed in the status quo equilibrium with  $R = R_0$ . We summarize the last set of results in the following lemma, which will be used later on in the characterization of the political equilibrium level of firing costs.

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<sup>19</sup>In particular, we remind that the terminal condition (10) implies the continuity of  $\mathcal{W} = \mathcal{W}(\cdot|R)$  at  $x = R$ , for any value of  $R$ .

**Lemma 2** *In voting between the two alternatives  $R_0$  and  $R'$ , where  $R' > R_0$ , the status quo is preferred by the employed workers with a level of idiosyncratic productivity  $x \in (x'_0, x^*)$  where  $x^*$  is defined by (26), and  $x'_0$  is defined by equation (27). All the unemployed, and the employed with productivity  $x \in [R_0, x'_0) \cup (x^*, \infty)$ , vote for the alternative  $R'$ .*

A particularly important implication of the analysis leading to Lemma 2 is the existence of a set of positive measure of workers, i.e. the employed in firms with productivity smaller than  $x^*$ , whose preferences over  $R$  are not single-peaked. The value of these workers is equal to  $\mathcal{W}(x|R)$  for any  $R$  such that  $\underline{R} \leq R \leq x$  and, by Lemma 1, it is strictly decreasing in  $R$  over the same range. However, the value of the same workers is equal to  $\mathcal{U}(R)$  (since they are fired) for any  $R$  such that  $x \leq R \leq 1$ , which as we know is strictly increasing in  $R$ , i.e. their preferences have two peaks. The presence of a set of positive measure of agents with non single-peaked preferences in the policy variable implies the violation of one of the assumptions of the median voter theorem, which therefore cannot be applied to solve for the political equilibrium of the model.

## 5 Political Equilibria

Before proceeding to characterize the political equilibria of the model, we introduce the following pair of definitions.

**Definition 1**  $R = R^*$  is a political equilibrium conditional on the *status quo*, if it defeats any other alternative in pairwise comparisons conditionally on  $R = R_0$ .

**Definition 2**  $R = R^*$  is an unconditional political equilibrium, if it defeats any other alternative in pairwise comparisons regardless on the *status quo* value of  $R$ .

We also define the threshold  $\hat{\beta}$  as the unique value of  $\beta$  solving the following equation (28)

$$[(1 - \beta) / (\beta \alpha^+)]^{\frac{1}{\alpha}} = 1, \quad (28)$$

that is, at  $\beta = \hat{\beta}$ , the threshold  $x^*$  defined by (26) is equal to the reservation productivity  $R = 1$  obtaining in absence of any firing costs.

**Proposition 2** *If  $\beta \leq \hat{\beta}$ , where  $\hat{\beta}$  is defined implicitly as the solution of (28), then the unique unconditional political equilibrium of the model involves setting  $R = 1$  (i.e.  $F = 0$ ), and this choice is preferred at unanimity to any alternative.*

**Proof.** See appendix. ■

Proposition 2 tells us that, as long as the rent extraction power of the employed is relatively low, a fully flexible labor market is politically stable, in the sense that workers prefer it at unanimity to *any* possible alternative, whatever the status quo is. The intuition for this result is that when the rents appropriated by the employed are small enough, workers have little reason to protect them by demanding any job security provisions, since the costs of employment protection are larger than the corresponding gains for any positive value of  $F$ .

To complete the characterization of the political equilibrium, let us define  $\bar{x}$  as the productivity level such that a worker is indifferent between being employed in the most rigid economy (i.e. where  $F = \hat{F}$ ) in a firm with idiosyncratic productivity equal to  $\bar{x}$ , and unemployed in the most flexible economy possible (i.e. where  $F = 0$ ). Formally,  $\bar{x}$  is defined implicitly by equation (27), setting  $R_0 = \underline{R}$  and  $R' = 1$ . Finally, let  $\lambda_\Psi \{(\cdot, \cdot)\}$  indicate the Lebesgue-Stieltjes measure induced by the distribution function of productivity across active firms  $\Psi(\cdot)$ , defined in Proposition 1.

**Proposition 3** *Suppose that  $\beta > \hat{\beta}$ , where  $\hat{\beta}$  is defined implicitly as the unique solution of (28). We have that that:*

1. *If  $R_0 < \bar{x}$ , then  $R = \underline{R}$  is the unique conditional political equilibrium if*

$$\lambda_\Psi \{(\bar{x}, x^*)\} L \geq \frac{1}{2}. \quad (29)$$

*Vice versa,  $R = 1$  is the unique conditional political equilibrium if the reverse of condition (29) holds.*

2. *If  $R_0 \geq \bar{x}$ , then  $R = \underline{R}$  is the unique conditional political equilibrium if*

$$\lambda_\Psi \{(R_0, x^*)\} L \geq \frac{1}{2}. \quad (30)$$

*Vice versa,  $R = 1$  is the unique political equilibrium if the reverse of condition (30) holds.*

**Proof.** See appendix. ■

Proposition 3 describes the basic structure of the political equilibrium, which has the following characteristics. First, according to Proposition 3 the political equilibrium always exists, and it involves either the choice of an unregulated labor market (i.e.  $F = 0$ ) or a maximally rigid labor market (i.e.  $F = \hat{F}$ ). Second, according to Proposition 3, labor market rigidity is supported by the workers who are employed in firms which have an intermediate

level of idiosyncratic productivity when voting occurs. Vice versa, flexibility is supported by an extreme coalition made up by the workers employed by the more and by the less productive firms, and also by all the unemployed. Third, Proposition 3 clarifies what role history plays in the model, i.e. how the political equilibrium emerging from the voting process is affected by the level of employment protection  $R_0$  present in the status quo.<sup>20</sup> In particular, according to Proposition 3, a set of positive measure of least productive employed workers favoring the transition to a highly flexible labor market exists provided that  $R_0 < \bar{x}$ , i.e. the economy is relatively rigid to begin with. Vice versa, if  $R_0 \geq \bar{x}$  all the employed workers with productivity lower than  $x^*$  are in favor of (more) labor market rigidity.

**Remark 3** If  $\beta = \hat{\beta}$ , equation (27) implies that  $x^* = \bar{x} = 1$ . Since  $\bar{x}$  is decreasing in  $\beta$ , and  $x^*$  is increasing  $\beta$  and equal to 1 when  $\beta = \hat{\beta}$ , we have that  $\bar{x} < x^*$  and  $R_0 < x^*$  for any  $\beta$  greater than  $\hat{\beta}$ . It follows that the sets of workers in favor of a rigid labor market contemplated by two cases of Proposition 3 are both non-empty.

## 5.1 Properties of the Political Equilibrium

In this subsection, we characterize how some endogenous elements of the equilibrium of the model, i.e. the thresholds  $x^*$ ,  $\bar{x}$  and  $\hat{\beta}$ , are affected by the key exogenous parameters.

We begin by observing that the threshold value  $x^*$  depends directly on the rent extraction power of the employed  $\beta$  and, through  $\alpha$ , on the drift  $\mu$  and on the instantaneous standard deviation  $\sigma$ . In particular, it can be shown with a straightforward differentiation of (26) that  $x^*$  is strictly increasing in  $\beta$ . This result is not surprising, since employment protection is attractive for the employed only if the rents that they capture, which are proportional to  $\beta$ , are large enough to compensate for the general equilibrium distortions caused by firing costs. Perhaps more surprisingly, how  $x^*$  is affected by the other parameters of interest,  $\sigma$  and  $\mu$ , depends on the rent extraction power of the employed, as clarified by the following lemma.

**Lemma 3** *Higher volatility and lower growth both increase the threshold  $x^*$  defined in (26) if  $\beta \in (\beta^*, 1)$  where  $\beta^* = e/(e + \alpha^+)$ . Higher volatility and lower growth both decrease  $x^*$  if  $\beta \in (0, \beta^*)$ , and do not affect  $x^*$  if  $\beta = \beta^*$ .*

**Proof.** It follows from the differentiation of expression (26). ■

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<sup>20</sup>The political equilibrium also depends on the status quo value of  $R$  through the level of employment  $L$ , which in depends on the level of firing costs. However, as we know from the analysis of the economic equilibrium firing costs have ambiguous effects on equilibrium employment since they reduce by firings and hirings.

In words, how employed workers are affected by greater volatility of output growth and by depressed productivity growth depends crucially on their power of rent extraction. Where this power is relatively high, i.e. if  $\beta > \beta^*$ , more volatility increases the productivity  $x^*$  of the marginal worker, as defined in Lemma 1, and vice versa. Intuitively, this is because of the following reason. It can be shown that in a more volatile economy, *both* the marginal gain and the marginal loss caused by more flexibility to the employed workers are *magnified*, reflecting the existence of a complementarity between flexibility and volatility. Which of these two opposite effects dominates over the other depends on  $\beta$ . If  $\beta > \beta^*$ , the rent is a relatively important component of the welfare of the employed, and therefore the magnification of the marginal loss dominates over that of the marginal gain from higher labor market flexibility. As a result, the threshold  $x^*$  has to increase in order to ensure that the marginal worker is *more insulated* from the risk of job destruction by a higher idiosyncratic productivity.<sup>21</sup> The opposite happens if the power of rent extraction of the employed is relatively small, i.e. if  $\beta < \beta^*$ , in which case the magnification of the marginal loss is dominated by the magnification of the marginal gain from more labor market flexibility and, as a result, the critical productivity level  $x^*$  has to fall. Similarly, it is possible to verify that *lower* productivity growth increases both the marginal gain and the marginal loss caused by more flexibility. When  $\beta$  is relatively high, the magnification of the marginal loss dominates, and therefore  $x^*$  must increase relative to its initial value in order to make the marginal worker more insulated from the risk of job destruction, reducing its marginal loss from more flexibility. The converse is true if  $\beta$  is relatively low, in which case the threshold  $x^*$  must decrease.

The next lemma clarifies how the second productivity threshold  $\bar{x}$  contemplated in Proposition 3 is affected by the parameters of interest.

**Lemma 4** *The productivity level  $\bar{x}$  defined in (27) with  $R_0 = \underline{R}$  and  $R' = 1$  increases with  $\sigma$  and decreases with  $\beta$  and  $\mu$ .*

**Proof.** See appendix. ■

That  $\bar{x}$  decreases with  $\beta$  is again not surprising since, if the employed workers are able to extract higher rents from firms, their position of insiders becomes clearly more appealing, and this creates more scope for the desire to protect it with job security provisions. As a

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<sup>21</sup>We remind that, because the paths of a Brownian process are (almost surely) continuous, the current productivity level of a firm exhibits some degree of persistence in the future. Therefore, the matches that are relatively productive in the present are exposed to a lower risk of destruction in the future, all else equal. This is reflected in fact that, as already remarked, the rent of the employed decreases less with  $R$  the greater is  $x$ , i.e. the cross-partial derivative of  $V$  with respect to  $x$  and  $R$  is strictly positive.



result, the productivity threshold  $\bar{x}$  falls. Higher volatility  $\sigma$  has qualitatively the opposite effect of  $\beta$  on the threshold  $\bar{x}$  because in a more volatile environment, the risk of a critical fall in productivity down to the absorbing barrier  $R$  is higher. As a result, the least productive workers expect to earn less rents, for any given level of firing costs, which makes them willing to give up, i.e.  $\bar{x}$  increases. Finally, a higher value of the drift coefficient  $\mu$  is found to reduce  $\bar{x}$ . Intuitively, a higher value of  $\mu$  means that employed workers, for any level of current idiosyncratic productivity, expect to become relatively more productive in future. In particular, a higher value of  $\mu$  induces the workers employed at the moment of voting in low productivity firms to become relatively more optimistic about their *future* productivity, and therefore about the future amount of rents that they can appropriated of, conditionally on remaining employed. The greater optimism makes these workers more reluctant to give up their position of insiders by voting in favor of less stringent job security provisions.<sup>22</sup>

Finally, Lemma 5 clarifies how the threshold value  $\hat{\beta}$  depends on the parameters governing the dynamics of productivity.

**Lemma 5** *The threshold  $\hat{\beta}$  defined implicitly by equation (28) increases with  $\sigma$  and it decreases with  $\mu$ .*

**Proof.** Straightforward implicit differentiation of equation (28). ■

According to Lemma 5 it is more likely, in the sense that the threshold  $\hat{\beta}$  shifts to the right, to obtain an equilibrium with unanimous political support for full flexibility (i.e.  $F = 0$ ) in a more volatile economic environment. Intuitively, it can be shown that a fully flexible labor market is demanded at unanimity if  $x^* \leq 1$ .<sup>23</sup> Moreover, as it can be easily verified by solving for  $\hat{\beta}$  in equation (28),  $\hat{\beta} < \beta^*$ , where  $\beta^*$  is defined as in Lemma 3. It then follows directly from Lemma 3 that when  $\beta = \hat{\beta}$  more volatility decreases the threshold productivity  $x^*$ , i.e. it enlarges the set of parameters values such that  $x^* \leq 1$ . In particular, since  $x^*$  is strictly increasing in  $\beta$ , the set of values of  $\beta$  consistent with a unanimous support for no firing costs becomes larger as  $\sigma$  increases. Also, by Lemma 3 the threshold  $x^*$  increases when  $\mu$  increases and, as a result, the set of values of  $\beta$  consistent with a unanimous support for no firing costs

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<sup>22</sup>The proof of Lemma 4 assumes that the status quo reservation productivity does not change as the parameters in question change, because of the assumption that people vote on the effective level of employment protection  $R$  rather than on the legislated one  $F$ . However, because the relation between  $R$  and  $F$  defined by equation (20) depends on these parameters,  $R$  itself would change if workers vote on  $F$ . Nonetheless, the result demonstrated in the next section on the base of Lemma 4, that there is not a clear-cut relation between changes in the economic environment, i.e. in  $\sigma$  and  $\mu$ , and the political viability of a reform of a rigid economy, does not depend on whether  $R$  is held constant or not.

<sup>23</sup>This is established in the proof of Proposition 2 reported in the appendix.

becomes smaller, i.e.  $\hat{\beta}$  decreases.

## 6 The Rise and Persistence of Eurosclerosis

In this section, we attempt to use our model in order to shed some new light on the comparative dynamics of labor market institutions in Continental Europe and in the U.S. and U.K. In particular, we are interested in the question of how some unexpected shocks to the main parameters of the model, i.e.  $\beta$ ,  $\sigma$  and  $\mu$ , affect the political equilibrium given the status quo level of firing costs. Before continuing, we remind that a useful property of the model previously remarked is that the value functions of all workers only depend on jump-variables, i.e. workers make their voting decisions “comparing steady states.” Moreover, the fact that voting takes place as soon as an unexpected shock to the exogenous parameters occurs, implies that the state-variables, i.e. the level of employment and the cross-sectional distribution of productivity, are not affected on impact by the shock, which affects the equilibrium of the model through the jump-variables only.

It is been widely remarked that the divergence of the labor market institutions of Continental Europe and of the U.S. has begun in the aftermath of the major negative macroeconomic shocks, increasing volatility and reducing productivity growth, occurred during the 1970's.<sup>24</sup> Our model is consistent with this observation, since it implies that institutional divergence can occur if the *same* negative shock hits economies which are relatively flexible to begin with, but which differ in terms of the ability of labor to appropriate rents. According to Proposition 3, in a relatively flexible economy (i.e. with  $R_0 \geq \bar{x}$ ) a transition to a rigid labor market is favored by the employed workers with productivity in the interval  $(R_0, x^*)$ , which has measure induced by  $\Psi(\cdot)$  equal to

$$\lambda_{\Psi} \{(R_0, x^*)\} L = [\Psi(x^*) - \Psi(R_0)] L. \quad (31)$$

The size of the coalition for rigidity increases with the rent extraction power of the employed  $\beta$  since, as we already know,  $\partial x^*/\partial \beta > 0$ . Moreover, according to Lemma 3, how the size of the coalition for rigidity is affected by a negative economic shock also depends on the value of  $\beta$ . In particular, in an economy where labor appropriates of relatively high rents, i.e. where  $\beta > \beta^*$ , the threshold productivity level  $x^*$  increases as  $\partial x^*/\partial \sigma > 0$  and  $\partial x^*/\partial \mu < 0$ . Vice versa, in an economy where labor appropriates of relatively low rents, i.e. where  $\beta < \beta^*$ , the threshold

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<sup>24</sup>See for example Gottschalk and Moffitt (1994) and Comin and Philippon (2005) for evidence of increased earnings and output volatility since 1970's.

productivity level  $x^*$  decreases in response to the same economic shock, as  $\partial x^*/\partial \sigma < 0$  and  $\partial x^*/\partial \mu > 0$ .

Since the threshold  $x^*$  is the only element of (31) affected by the shock in question, we conclude that “bad times,” such as those experienced by virtually all industrialized economies during much of the 1970’s, increase the political support for the transition to a more rigid labor market in high rents economies, such as those of Continental Europe. However, the same type of shock does not make labor market rigidity more appealing politically in low rents economies, such as the U.S., where indeed the size of the coalition for rigidity shrinks. This result depends on the complementarity existing between volatility and flexibility (and on substitutability existing between growth and flexibility), i.e. more volatility (or less growth) boosts simultaneously the positive effect of flexibility on job creation, as well as its negative effect on the rents of the employed. It follows that employed workers demand higher firing costs only if the rent is a relatively important component of their welfare, i.e. if their rent extraction power is high enough.

At the present moment, the political debate over labor market institutions in Continental Europe is centered around the elimination of part of their rigidity. According to some authors (e.g. Bean, 1998) “good times” should be a favorable period for implementing reforms. Our model, vice versa, calls into question this proposition since it implies the inexistence of a simple relation between macroeconomic well-being and the political feasibility of labor market reforms toward more flexibility. This is because, as explained in the following, good times make flexibility more appealing among the most productive employed but, in a relatively rigid economy, they also make it less appealing among the less productive employed, whose job is sheltered by the high status quo level of job security provisions.

As we know, according to Proposition 3 the reform of a relatively rigid economy, i.e. with  $R_0 < \bar{x}$ , consisting in increasing labor market flexibility, is supported by all the unemployed, and by the employed with productivity in the upper and lower tail of the distribution, i.e. with  $x \in [R_0, \bar{x})$  and with  $x \in (x^*, \infty)$ . A higher value of  $\beta$  decreases without ambiguity the size of the coalition for reform, since  $x^*$  is strictly increasing and  $\bar{x}$  is strictly decreasing in  $\beta$  and, as a result, the size of the coalition for the preservation of the rigid status quo also increases if the employed workers extract more rents from the firms. To determine what impact  $\sigma$  and  $\mu$  have on the two threshold levels of productivity  $x^*$  and  $\bar{x}$ , we need to distinguish between the two different cases contemplated by Lemma 3, corresponding respectively to values of  $\beta$  smaller or greater than the threshold  $\beta^*$ . The case of  $\beta > \beta^*$  is more relevant here since it reflects the

high-rents economies of Continental Europe. If  $\beta > \beta^*$  Lemmas 3 and 4 imply that the effects of  $\sigma$  and  $\mu$  on  $x^*$  are such that  $\partial x^*/\partial\sigma > 0$  and  $\partial x^*/\partial\mu < 0$ , and their effects on  $\bar{x}$  are such that  $\partial\bar{x}/\partial\sigma > 0$  and  $\partial\bar{x}/\partial\mu < 0$ .<sup>25</sup>

What can the model tell us about how favorable macroeconomic conditions affect the political viability of reforms of European labor markets? Unfortunately, not a clear-cut message. This is because, as we know, the two thresholds  $x^*$  and  $\bar{x}$  defining the coalition for rigidity tend to move in the same direction, i.e. they both decrease, if  $\sigma$  decreases and if  $\mu$  increases. In particular, the measure of the set  $(x^*, \infty)$  of most productive employed workers who stand for flexibility, or

$$\lambda_{\Psi} \{(x^*, \infty)\} L = [1 - \Psi(x^*)] L,$$

increases as  $x^*$  decreases. However, the same type of shock also tends to cut down the political support for flexibility among the workers located at the bottom of the distribution of productivity, namely the measure of the set  $(R_0, \bar{x})$

$$\lambda_{\Psi} \{(R_0, \bar{x})\} L = [\Psi(\bar{x}) - \Psi(R_0)] L,$$

by moving out of it some workers with productivity below the lower bound  $\bar{x}$ . Intuitively, this is because good economic conditions boost the rents that employed workers can potentially obtain, and therefore make some of them more reluctant to give up their position of insiders. As a result, whether the extent of the political support for rigidity among the workers with intermediate productivity, i.e. the measure of the set  $(\bar{x}, x^*)$ , increases or decreases cannot be established *a priori*.

This result is particularly important, since it implies that the way labor market institutions evolve in response to a worsening and to an improvement of aggregate business conditions respectively, may be strikingly *asymmetric*. In particular, whereas a bad economic shock may cause the breakdown of a relatively flexible economy, a good shock hitting a rigid high-rents economy need *not* have the effect of triggering the opposite transition to a more flexible labor market. This result is broadly consistent with, and provides a novel explanation for the dynamics of labor market institutions observed in Continental Europe in the recent years, which have shown little tendency to revert to flexibility, long after the original negative shocks favoring the build-up of Eurosclerosis have vanished.<sup>26</sup>

<sup>25</sup>If instead  $\beta < \beta^*$ , Lemmas 3 and 4 imply that  $\partial x^*/\partial\sigma < 0$  and  $\partial x^*/\partial\mu > 0$  and, again,  $\partial\bar{x}/\partial\sigma > 0$  and  $\partial\bar{x}/\partial\mu < 0$ . As a result, the coalition for rigidity in a low-rents economy becomes actually larger if the economy is hit by a good economic shock. This result is analogous to Lemma 5, which implies that an improvement of economic conditions makes it more unlikely to have a unanimous support for labor market flexibility.

<sup>26</sup>It must be emphasized that the persistence of high levels of rent extraction power on the part of employed

We conclude this section by remarking that our model questions the validity of the argument that good times are necessarily good also for reforms. Cutting down the rents appropriated by the employed, i.e. reducing the value of  $\beta$ , is an important pre-requisite of a successful reform of a rigid labor market. However, favorable economic shocks do not have clear-cut consequences for the political feasibility of a reform aimed at making a rigid labor market more flexible.

## 7 Conclusions

This paper has presented a simple model of political economy of the labor market, with the aim of explaining the comparative dynamics of one institution, employment protection legislation, in Europe and in the U.S. in the last few decades. At the methodological level, the paper represents an important innovation to the existing literature, since it relies on the novel assumption that the dynamics of productivity is described by a Geometric Brownian process rather than, as usually assumed, by a Poisson process. This assumption is important since it implies that the preferences on employment protection legislation of the workers are affected by their own idiosyncratic productivity at the moment of voting. This is because relatively more productive jobs are expected to last longer, due to the persistence of idiosyncratic productivity implied by the continuity of the paths of Brownian motion.

A key result demonstrated concerns the broad importance of the rents that employed workers are able to extract from firms. The capacity of labor to appropriate rents and labor turnover restrictions, have appeared to be closely linked as part of rigid politico-economic equilibria. This result is quite intuitive given that if rents are low, then there is clearly little scope to demand their protection with stringent job security provisions either. Moreover, and perhaps more surprisingly, how labor market institutions are affected by economic shocks, has also been found to depend on the extent of the rents appropriated by labor (i.e. on whether  $\beta$  is above the critical threshold  $\beta^*$  defined in Lemma 3), as well as on the status quo level of firing costs (i.e. on whether  $R_0$  is greater than  $\bar{x}$  according to Proposition 3).

The results provided by the analysis of the political equilibrium of the model, have then been used to demonstrate that the different bargaining strength of labor can explain the di-

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workers, is important according to our model to explain the lack of reversibility displayed by Continental European labor market institutions. This is documented empirically by Saint-Paul (2004), who finds no evidence of a decline in the rents of employed workers in Europe during the 1990's, with the exception of Ireland. See also Möller and Aldashev (2005), who document that employed workers have been able to appropriate of persistently higher rents in Germany than in the U.S., since the early 1980's.

verging pattern of institutional evolution experienced by Continental Europe and by the U.S., in response to similar major negative shocks during the 1970's. In addition, a novel potential explanation has been provided of why the institutional rigidity typical of Continental European labor markets, emerged in the past decades, has largely persisted up to the present day, long after the shocks originally favoring its creation have vanished. More generally, an important implication of our model is that once stringent job security provisions are put in place, they have the potential to be persistent across different economic conditions, i.e. there exists in this respect a scope for institutional hysteresis.

## 8 Appendix

### 8.1 Derivation of the Expected PDV of Output

The integral in (9) can be broken down recursively to obtain the following recursion, satisfied by the functional  $V(\cdot)$  over the region  $(R, \infty)$  of productivity levels such that firms continue operating

$$rV(x) = x + \frac{1}{dt}\mathbb{E}(dV). \quad (32)$$

The second order differential equation obtained by applying Ito's lemma to (32) in order to compute the expression of  $\mathbb{E}(dV)$ , reads

$$\frac{1}{2}\sigma^2 x^2 V''(x) + \mu x V'(x) - rV(x) + x = 0. \quad (33)$$

The general integral of this equation is represented by

$$V(x) = \frac{x}{r - \mu} + D_1 x^\nu + D_2 x^\alpha, \quad (34)$$

where  $\nu$  and  $\alpha$  denote respectively the positive and negative root of the relevant characteristic polynomial associated with (33), identical to the corresponding characteristic polynomial  $[(\sigma^2 \epsilon^2 / 2) + \mu \epsilon - r]$  associated with (41). The expression of equation (12) follows from (34), excluding the positive root  $\nu$  by setting  $D_1 = 0$  in (34) for the standard reason (e.g. Dixit, 1993, p. 25), i.e. to prevent the fundamental of the asset to become negligible relatively to its option value as  $x \uparrow \infty$ , and taking into account the boundary condition  $V(R) = 0$ .

### 8.2 Derivation of the Wage Schedule

Combining equations (8) and (9) by setting  $x = 1$ , the flow-value of unemployment can be expressed as

$$r\mathcal{U} = b + \theta\beta V(1). \quad (35)$$

Substituting for  $\mathcal{W}(x)$  using equation (9), and using the fact, also implied by (9), that  $\mathbb{E}(d\mathcal{W})/dt = \beta\mathbb{E}(dV)/dt$ , the recursion (7) can be written as

$$r\mathcal{U} + r\beta V(x) = w(x) + \beta \frac{1}{dt} \mathbb{E}(dV), \quad (36)$$

Also, by combining equations (35) and (36), we obtain that

$$w(x) = b + \theta\beta V(1) + r\beta V(x) - \beta \frac{1}{dt} \mathbb{E}(dV). \quad (37)$$

Finally, substituting in this equation the expression of  $V(x)$  provided by (32), (37) can be written as the expression reported in equation (11).

### 8.3 Solution of the Firms' Optimal Stopping Problem

Firms face a standard problem of optimal stopping in continuous time, which is formalized by the Bellman-Wald equation (4). It is well known that the solution of this class of problems (e.g. Dixit, 1993; Dixit and Pindyck, 1994; Peskir and Shiryaev, 2006) is characterized in terms of a productivity threshold  $R$ , such that the continuation value of the asset exceeds the value of the asset upon stopping as long as  $x > R$  and is exceeded by it if  $x < R$ , with the two values matching at  $x = R$ . The optimal stopping rule of the firm is to continue producing as long as  $x$  remains above  $R$ , and to close down, firing the workers and paying the associated layoff cost  $F$ , as soon as the absorbing barrier  $R$  is first reached. On the continuation region  $\{x \in \mathbb{R}_+ : x > R\}$ , therefore, the functional equation (4) corresponds to the Hamilton-Jacobi-Bellman equation

$$r\mathcal{J}(x) = x - w(x) + \frac{1}{dt} \mathbb{E}(d\mathcal{J}), \quad (38)$$

while at the absorbing barrier  $R$ , the following “value matching” (or continuous fit) condition

$$\mathcal{J}(R) = -F \quad (39)$$

must hold, establishing the continuity of the value function  $\mathcal{J}(\cdot)$  upon stopping. A second functional relation, the “smooth pasting” (or smooth fit) condition, must also hold for the stopping rule to be optimal. This condition states that the value function is differentiable with continuity along the curve separating the continuation region from the stopping region. Here,

the continuation value of the firm upon stopping is equal to  $-F$ , and therefore the smooth pasting condition implies that

$$\mathcal{J}'(R) = 0. \quad (40)$$

Equation (38) can be transformed into a second order ordinary differential equation in the unknown function  $\mathcal{J}(\cdot)$  by applying Ito's lemma to compute the expression of  $\mathbb{E}(d\mathcal{J})$ . This allows us to transform the optimal stopping problem of the firm in a free-boundary problem. The differential equation satisfied by  $\mathcal{J}(\cdot)$  reads

$$\frac{1}{2}\sigma^2 x^2 \mathcal{J}''(x) + \mu x \mathcal{J}'(x) - r \mathcal{J}(x) + x - w(x) = 0. \quad (41)$$

Using the expression of the wage rate  $w(x)$  reported in (11), the general integral of this equation is found to be of the type

$$\mathcal{J}(x) = \frac{(1-\beta)x}{r-\mu} - \frac{b+\theta\beta V(1)}{r} + D_1 x^\varrho + D_2 x^\alpha,$$

with  $D_1$  and  $D_2$  standing for constants to be determined, and with  $\varrho$  and  $\alpha$  standing for the positive and for the negative root of the characteristic polynomial

$$\frac{\sigma^2}{2}\epsilon(1-\epsilon) + \mu\epsilon - r \quad (42)$$

associated with (41). By a standard argument (e.g. Dixit, 1993, p. 25), the root  $\varrho$  must be eliminated by setting the constant  $D_1$  equal to zero. This is because otherwise the fundamental of the asset would become negligible, relatively to its option value, as  $x \uparrow \infty$ . The value of  $D_2$  is instead determined through the smooth pasting condition, which implies that

$$D_2 = \frac{(1-\beta)R^{1-\alpha}}{(r-\mu)\alpha^+}.$$

The expression of equation (13) then follows. Moreover, since the negative root  $\alpha$  of (42) reads

$$\alpha = \frac{1}{\sigma^2} \left( \frac{\sigma^2}{2} - \mu - \frac{1}{2} \sqrt{(\sigma^2 - 2\mu)^2 + 8\sigma^2 r} \right), \quad (43)$$

it can be verified with straightforward algebra that

$$\frac{\partial \alpha}{\partial \sigma} > 0 \text{ and } \frac{\partial \alpha}{\partial \mu} < 0. \quad (44)$$



## 8.4 Proof of Proposition 1

In order to describe the evolution of the productivity of a firm it is convenient to consider, rather than the original process  $x$ , the transformed process  $z \equiv \ln x$ . It is known (e.g. Dixit, 1993) that, since  $x$  represents a Geometric Brownian process with drift  $\mu$  and instantaneous standard deviation  $\sigma$ ,  $z$  is a linear Brownian process with mean  $\eta = [\mu - (\sigma^2/2)]$ , and instantaneous standard deviation  $\sigma$ . Notice that, because the initial value of  $x$  is normalized to one, the initial value of  $z$  is equal to zero. Moreover, the drift  $\eta$  of the transformed process is negative, since  $\mu < \sigma^2/2$  by assumption.

Next, focusing the attention with no loss of generality on a firm created at time  $s = 0$ , define  $p(z_0, z; t)$  as the probability density function of  $z$ , conditional on the fact that and that the process  $z$  has never reached the barrier  $\hat{R} \equiv \ln R$  within the time interval  $(0, t)$ , starting at  $z(0) \equiv z_0$ . We can write the conditional distribution function corresponding to the density  $p(z_0, z; t)$  as

$$\Pr \left\{ z(t) > z \mid z(\tau) > \hat{R}, \forall \tau \in (0, t) \right\} = \int_z^\infty p(z_0, \zeta; t) d\zeta \equiv P(z_0, z; t). \quad (45)$$

Using this expression, we can write the probability that the process  $z$  has not yet been absorbed at  $\hat{R}$  up to time  $t$ , as  $P(z_0, \hat{R}; t)$ . Moreover, defining  $T(\hat{R})$  as the random time elapsed since the creation of the firm in question, at which the process describing the evolution of the (log of) its productivity first reaches the barrier  $\hat{R}$ , we obviously have that

$$\Pr \left\{ T(\hat{R}) > t \right\} = P(z_0, \hat{R}; t). \quad (46)$$

Since our objective is to compute the probability distribution of a first passage time of a Brownian motion, it is natural to look at the Kolmogorov backward partial differential equation satisfied its transition density. It is known (e.g. Cox and Miller, 1965, ch. 5) that the function  $P(z_0, \hat{R}; t)$  satisfies the Kolmogorov backward partial differential equation, and therefore we can write that

$$\frac{1}{2}\sigma^2 \frac{\partial^2 P}{\partial z_0^2} + \eta \frac{\partial P}{\partial z_0} = \frac{\partial P}{\partial t}, \quad (47)$$

given the pair of boundary conditions

$$P(\hat{R}, \hat{R}; t) = 0 \text{ and } \lim_{z_0 \rightarrow \infty} P(z_0, \hat{R}; t) = 1. \quad (48)$$

The first boundary condition reflects the fact that absorption immediately occurs if  $z_0 = \hat{R}$ , and the second boundary condition reflects the fact that absorption occurs with probability

zero in a finite time, if the process  $z$  starts at an initial position infinitely distant from the barrier. For our purpose, it is convenient to solve the boundary value problem represented by (47) and (48) with the Laplace transform method. Let  $\mathcal{L}(\cdot; z_0, \hat{R})$  indicate the Laplace transform of  $P(z_0, \hat{R}; t)$ , defined as

$$\mathcal{L}(\rho; z_0, \hat{R}) = \int_0^\infty e^{-\rho t} P(z_0, \hat{R}; t) dt. \quad (49)$$

By transforming both sides of equation (47) using the theorem of differentiation of the original, and the fact that at the moment of creation, the probability that the productivity of a firm is equal to  $R$  is zero, i.e.

$$P(z_0, \hat{R}; 0) = 1,$$

it can be shown that  $\mathcal{L}(\rho; \cdot, \hat{R})$  satisfies the following second order ordinary differential equation

$$\frac{1}{2}\sigma^2 \frac{d^2 \mathcal{L}}{dz_0^2} + \eta \frac{d\mathcal{L}}{dz_0} = \rho \mathcal{L} - 1, \quad (50)$$

subject to the pair of transformed boundary conditions

$$\mathcal{L}(\rho; \hat{R}, \hat{R}) = 0 \text{ and } \lim_{z_0 \rightarrow \infty} \mathcal{L}(\rho; z_0, \hat{R}) = \frac{1}{\rho}. \quad (51)$$

Letting

$$\vartheta(\rho) = \frac{-\eta - \sqrt{\eta^2 + 2\sigma^2\rho}}{\sigma^2}$$

denote the negative root of the characteristic polynomial  $[(\sigma^2\epsilon^2/2) + \eta\epsilon - \rho]$  associated with (50), as a function of  $\rho$ , the solution of equation (50) subject to (51), is found to be

$$\mathcal{L}(\rho; z_0, \hat{R}) = \frac{1}{\rho} \left[ 1 - e^{\vartheta(\rho)(z_0 - \hat{R})} \right]. \quad (52)$$

Since  $\eta < 0$ , and since  $z_0 = 0$ , the following pair of equalities holds

$$\mathcal{L}(0; 0, \hat{R}) = \lim_{\rho \rightarrow 0} \frac{1}{\rho} \left[ 1 - e^{-\vartheta(\rho)\hat{R}} \right] = \frac{\hat{R}}{\eta}, \quad (53)$$

where the second equality follows by applying de l'Hospital theorem to compute the limit. Finally, using (53), and the fact that (46) and (49) imply that

$$\mathcal{L}(0; 0, \hat{R}) = \int_0^\infty \Pr\{T(R) > t\} dt, \quad (54)$$

the steady state rate of aggregate job destruction  $\delta$ , characterized by (16), can be expressed in the closed-form

$$\delta = \frac{1}{\mathcal{L}(0; 0, \hat{R})} = \frac{1}{\hat{R}^+} \left( \frac{\sigma^2}{2} - \mu \right),$$

which corresponds to the expression reported in (22).

To complete the proof of Proposition 1, we need to characterize the ergodic cross-sectional distribution of productivity across active firms. Using again the transformation  $z \equiv \ln x$ , we can write the the steady state cross-sectional distribution of  $z$  across employment as

$$\tilde{\Psi}(z) \equiv \Pr\{Z \leq z\} = 1 - \Pr\{Z > z\}. \quad (55)$$

Also, using the expression of the transition density of  $z$  conditional on non-absorption defined in (45), we can write that

$$\Pr\{Z > z\} = \frac{\theta(1-L)}{L} \int_{-\infty}^t \left[ \int_z^{\infty} p(z_0, \zeta; t-s) d\zeta \right] ds.$$

This expression corresponds to the integral sum of the number of firms created since the infinitely remote past, which have survived up to time  $t$  and have productivity at  $t$  greater than  $z$ , weighted by the steady state level of employment.

Next, using the expression of  $\delta$  derived above and reported in (22), the fact that equation (15) implies that  $\theta(1-L)/L = \delta$ , and changing variables, we can also write this expression as

$$\Pr\{Z > z\} = \frac{\eta}{\hat{R}} \int_0^{\infty} P(z_0, z; t) dt, \quad (56)$$

where  $P(z_0, z; t)$  is defined as in (45). Using (56) we can then write (55) as

$$\tilde{\Psi}(z) = 1 - \frac{\eta}{\hat{R}} \int_0^{\infty} P(z_0, z; t) dt. \quad (57)$$

To make progress in characterizing  $\tilde{\Psi}(\cdot)$ , we consider the Kolmogorov backward differential equation satisfied by  $P(z_0, z; t)$ , which is equivalent to equation (47), together with the pair of boundary conditions, which have the usual interpretation,

$$P(\hat{R}, z; t) = 0 \text{ and } \lim_{z_0 \rightarrow \infty} P(z_0, z; t) = 1. \quad (58)$$

It is again convenient to solve the backward equation (47) subject to (58), with the Laplace transform method. Defining the Laplace transform of the function  $P(z_0, z; t)$  as

$$\mathcal{L}(\rho; z_0, z) = \int_0^{\infty} e^{-\rho t} P(z_0, z; t) dt, \quad (59)$$

the expression of  $\tilde{\Psi}(\cdot)$  can be directly obtained by computing the limit of  $\mathcal{L}(\rho; z_0, z)$  as  $\rho \downarrow 0$ . To compute the expression of the Laplace transform (59), we begin by transforming equation

(47) in the following pair of second order ordinary differential equations,

$$\frac{1}{2}\sigma^2 \frac{d^2 \mathcal{L}}{dz_0^2} + \eta \frac{d\mathcal{L}}{dz_0} = \rho \mathcal{L} - 1, \text{ if } z_0 > z, \quad (60)$$

and

$$\frac{1}{2}\sigma^2 \frac{d^2 \mathcal{L}}{dz_0^2} + \eta \frac{d\mathcal{L}}{dz_0} = \rho \mathcal{L}, \text{ if } z_0 \leq z, \quad (61)$$

where  $\mathcal{L} = \mathcal{L}(\rho; z_0, z)$ , along with the pair of transformed boundary conditions obtained from (58)

$$\mathcal{L}(\rho; \hat{R}, z) = 0, \text{ and } \lim_{z_0 \rightarrow \infty} \mathcal{L}(\rho; z_0, z) = \frac{1}{\rho}. \quad (62)$$

Some tedious but simple algebra<sup>27</sup> implies that the solution to (60) and (61) subject to (62) is

$$\mathcal{L}(\rho; z_0, z) = \frac{\lambda_2}{\rho} e^{\lambda_1(\hat{R}-z)} \frac{e^{\lambda_1(z_0-\hat{R})} - e^{\lambda_2(z_0-\hat{R})}}{\lambda_2 - \lambda_1}, \text{ if } z_0 \leq z, \quad (63)$$

$$\mathcal{L}(\rho; z_0, z) = \frac{\lambda_2}{\rho} e^{\lambda_1(\hat{R}-z)} \frac{e^{\lambda_1(z_0-\hat{R})} - e^{\lambda_2(z_0-\hat{R})}}{\lambda_2 - \lambda_1} + \frac{1}{\rho} \left[ 1 - \frac{\lambda_2 e^{\lambda_1(z_0-z)} - \lambda_1 e^{\lambda_2(z_0-z)}}{\lambda_2 - \lambda_1} \right], \text{ if } z_0 > z, \quad (64)$$

where we have used the definitions

$$\lambda_1 \equiv -\frac{\eta}{\sigma^2} + \sqrt{\frac{\eta^2}{\sigma^4} + \frac{2\rho}{\sigma^2}} \text{ and } \lambda_2 \equiv -\frac{\eta}{\sigma^2} - \sqrt{\frac{\eta^2}{\sigma^4} + \frac{2\rho}{\sigma^2}}.$$

Computing the limit as  $\rho \downarrow 0$  of expressions (63) and (64) using de l'Hospital theorem, we obtain that

$$\mathcal{L}(0; z_0, z) = \frac{\sigma^2}{2\eta^2} \left\{ e^{-\frac{2\eta}{\sigma^2}(z_0-z)} \cdot \mathcal{I}_{\{R \leq z_0 \leq z\}} + \left[ 1 - \frac{2\eta}{\sigma^2}(z_0 - z) \right] \cdot \mathcal{I}_{\{z \leq z_0\}} - e^{-\frac{2\eta}{\sigma^2}(\hat{R}-z)} \right\}. \quad (65)$$

The ergodic cross-sectional distribution of  $z$  across active firms can at this point be computed by setting  $z_0 = 0$  in (65) and by substituting the corresponding expression in (57). Finally, the ergodic cross-sectional distribution of  $x$  across active firms  $\Psi(\cdot)$  can be simply obtained from the corresponding expression of  $z$  by setting  $z = \ln x$ , and it reads

$$\Psi(x) = 1 - \frac{\sigma^2}{2\eta \hat{R}} \left\{ \left[ \frac{2\eta}{\sigma^2} \ln(x) + 1 \right] \cdot \mathcal{I}_{\{R \leq x \leq 1\}} + e^{\frac{2\eta}{\sigma^2} \ln(x)} \cdot \mathcal{I}_{\{1 \leq x\}} - e^{-\frac{2\eta}{\sigma^2}[\hat{R}-\ln(x)]} \right\}. \quad (66)$$

The expression of the ergodic cross-sectional density function of productivity reported in (23) is obtained by differentiating  $\Psi(\cdot)$  with respect to  $x$  and rearranging terms.

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<sup>27</sup>The details of the following algebraic derivations are available upon request from the author.

## 8.5 Comparative Statics Properties of the Economic Equilibrium

Properties (1) and (2) follow from a straightforward implicit differentiation in equations (17) and (20).

To prove property (3), we begin by noticing that equation (20) implies that the equilibrium reservation productivity  $R$  depends on  $\sigma$  only through  $\alpha$ . Differentiating implicitly the equilibrium reservation productivity  $R$  with respect to  $\alpha$  in equation (20) we obtain that

$$\frac{dR}{d\alpha} = R \frac{1 - R^\alpha + \ln(R^\alpha)}{\alpha(1 - \alpha)(1 - R^\alpha)}, \quad (67)$$

and this expression is negative since the denominator of (67) is positive, while the numerator is negative. The denominator of (67) is positive since  $\alpha < 0$  and since equation (20) implies that  $R < 1$  for any positive value of  $F$ , so that  $R^\alpha > 1$ ; the fact that  $1 - a + \ln a < 0$  for every  $a \neq 1$  also implies that the numerator of (67) is negative. The result that in equilibrium  $R$  decreases with  $\sigma$  follows since  $\partial\alpha/\partial\sigma > 0$  by (44).

To prove property (4), it is useful to establish first the following preliminary result. Letting as before  $R$  denote the equilibrium reservation productivity defined by equation (20), using the expression of  $dR/d\alpha$  provided by (67) and rearranging terms, we have that

$$\frac{d(R^{1-\alpha})}{d\alpha} = R^{1-\alpha} \frac{1 - R^\alpha + R^\alpha \ln(R^\alpha)}{\alpha(1 - R^\alpha)} > 0, \quad (68)$$

since, as already remarked,  $R^\alpha > 1$  and moreover  $1 - a + a \ln(a) > 0$  for any  $a > 1$ .

Observe next that the equilibrium job creation rate  $\theta$  defined by (19) depends on  $\sigma$  only through  $\alpha$ , and that  $\alpha$  affects  $\theta$  both directly and indirectly through  $R$  (which as we already know by (20) depends on  $\sigma$  only through  $\alpha$ ). It follows that how  $\sigma$  affects  $\theta$  depends on the total derivative of  $\theta$  with respect to  $\alpha$ . By equation (19), the total derivative with respect to  $\alpha$  of the schedule  $\theta \equiv \theta(R(\alpha), \alpha)$  representing the equilibrium job creation rate as a function of  $\alpha$  can be written as

$$\frac{d\theta}{d\alpha} = \frac{[r(1 - \beta) + \alpha^+ \beta \theta] \frac{d(R^{1-\alpha})}{d\alpha} + [(r - \mu)b - r(1 - \beta)] + \theta \beta (1 - R^{1-\alpha})}{\alpha^+ \beta (1 - R^{1-\alpha})}. \quad (69)$$

Since equation (19) can also be written as

$$\theta \beta (1 - R^{1-\alpha}) = \frac{r(1 - \beta) R^{1-\alpha}}{\alpha^+} - [(r - \mu)b - r(1 - \beta)], \quad (70)$$

combining (69) and (70) we also have that

$$\frac{d\theta}{d\alpha} = \frac{r(1-\beta) + \alpha^+ \beta \theta}{\alpha^+ \beta (1 - R^{1-\alpha})} \frac{d(R^{1-\alpha})}{d\alpha} + \frac{r(1-\beta) R^{1-\alpha}}{\alpha^2 \beta (1 - R^{1-\alpha})}, \quad (71)$$

which is positive since as we already know by (68)  $d(R^{1-\alpha})/d\alpha > 0$  and obviously  $R^{1-\alpha} < 1$ . The result that in equilibrium  $\theta$  increase strictly with  $\sigma$  follows since  $\partial\alpha/\partial\sigma > 0$  by (44).

Finally, property (5) can be demonstrated by observing that the equilibrium reservation productivity  $R$  determined by equation (20) depends on  $\mu$  both directly and through  $\alpha$ , i.e. we have that

$$R \equiv R(\alpha(\mu), \mu). \quad (72)$$

The total derivative of this expression with respect to  $\mu$  can be represented as

$$\frac{dR}{d\mu} = \frac{\partial R}{\partial \alpha} \frac{\partial \alpha}{\partial \mu} + \frac{\partial R}{\partial \mu}.$$

A straightforward implicit differentiation of (20) implies that  $\partial R/\partial \mu > 0$ , i.e. holding  $\alpha$  constant,  $R$  increases with  $\mu$ . Moreover, we know from (67) that  $R$  is strictly decreasing in  $\alpha$  and we know from (44) that  $\partial\alpha/\partial\mu < 0$ . It follows that  $dR/d\mu > 0$ .

We can write the job destruction rate as a function of  $\mu$  using (22) and (72) as

$$\delta \equiv \delta(R(\alpha(\mu), \mu), \mu).$$

The total derivative of this expression with respect to  $\mu$  can be expressed as

$$\frac{d\delta}{d\mu} = \frac{\partial \delta}{\partial R} \frac{dR}{d\mu} + \frac{\partial \delta}{\partial \mu}.$$

A straightforward differentiation of equation (22) shows that the direct effect of  $\mu$  on  $\delta$  is negative, i.e.  $\partial\delta/\partial\mu < 0$ , and also that  $\delta$  increases with  $R$ , i.e.  $\partial\delta/\partial R > 0$ . Since as just demonstrated  $R$  is overall an increasing function of  $\mu$ , i.e.  $dR/d\mu > 0$ , we conclude that the sign of  $d\delta/d\mu$  is ambiguous.

To understand why productivity growth has a ambiguous effect on job creation, notice that  $\theta$  depends on  $\mu$  in a variety of ways. In particular,  $\mu$  affects  $\theta$  directly, but also indirectly through  $\alpha$  and through the equilibrium reservation productivity (itself a function of  $\alpha$  and  $\mu$ ). Using (19) and (72), we can write the equilibrium job creation rate as a function of  $\mu$  as

$$\theta \equiv \theta(R(\alpha(\mu), \mu), \alpha(\mu), \mu).$$

The total derivative of this expression with respect to  $\mu$  can be expressed as

$$\frac{d\theta}{d\mu} = \frac{\partial \theta}{\partial \alpha} \frac{\partial \alpha}{\partial \mu} + \frac{\partial \theta}{\partial R} \frac{dR}{d\mu} + \frac{\partial \theta}{\partial \mu}. \quad (73)$$

A straightforward differentiation of equation (19), that the direct effect of  $\mu$  on  $\theta$  is positive, i.e.  $\partial\theta/\partial\mu > 0$ . Moreover, we know that by (19) in equilibrium  $\theta$  increases with  $R$ , i.e.  $\partial\theta/\partial R > 0$ , and that  $R$  increases with  $\mu$ , i.e.  $dR/d\mu > 0$ . This implies that both the second and the term component of the total derivative of  $\theta$  with respect to  $\mu$  (73) are positive. However, by (71), we know that  $\alpha$  has a positive direct effect on  $\theta$ , i.e.  $\partial\theta/\partial\alpha > 0$ , and that  $\partial\alpha/\partial\mu < 0$ , which means that the first term of  $d\theta/d\mu$  is negative. Which of the opposite effects of  $\mu$  on  $\theta$  dominates over the other cannot be established *a priori*.

## 8.6 Proof of Proposition 2

We already know that the value of the unemployed is strictly increasing in  $R$  for any value of  $R$ , and for any value of  $\beta$ . To determine the voting decision of the employed workers under the assumption stated in Proposition 2 that  $\beta \leq \hat{\beta}$ , where we remind that  $\hat{\beta}$  is defined as the (unique) solution to equation  $x^*(\beta) = 1$ , we begin by noticing that, since the threshold  $x^*$  defined in (26) is strictly increasing in  $\beta$ , this condition implies that

$$x^* \leq 1, \quad (74)$$

namely that  $x^*$  is lower or equal to the reservation productivity obtaining when  $F = 0$ . Some simple algebra shows that (74) implies that

$$x^* - (R)^{1-\alpha} (x^*) \leq (x^*)^\alpha \left[ 1 - (R)^{1-\alpha} \right],$$

for *any* possible value of  $R$ , which in turn implies that

$$\mathcal{W}(x^* | R) \leq \mathcal{U}(1), \quad (75)$$

where we remind that  $\mathcal{W}(x | R)$  denotes the value of employment in a firm with productivity  $x$ , conditionally on  $R$  expressed by equation (25), and  $\mathcal{U}(1)$  denotes the value of unemployment conditionally on  $R = 1$  (i.e. on  $F = 0$ ) expressed by equation (24).

In particular, because  $\mathcal{W}(\cdot | R)$  is strictly increasing in  $x$ , if condition (75) holds, all the workers who are employed in the status quo in firms with productivity  $x \in [R_0, x^*]$ , and whose value is  $\mathcal{W}(x | R_0)$  in the status quo and  $\mathcal{W}(x | R)$  for any  $R$  such that  $R \leq R_0$ , are better-off as unemployed with  $R = 1$ , than they are as employed for any value of  $R$  such that  $R \leq R_0$ . If instead the reservation productivity is set at any level  $R$  such that  $R > R_0$ , a worker with productivity  $x \in [R_0, x^*]$  remains employed if  $x > R$ , in which cases its welfare decreases relative to  $\mathcal{W}(x | R_0)$  as the labor market becomes more flexible by Lemma 1, and

it is therefore also lower than  $\mathcal{U}(1)$  by (75), or becomes unemployed if  $x \leq R$ . If this case, its welfare is as well lower than  $\mathcal{U}(1)$ , since the value of unemployment is strictly increasing in  $R$  for any value of  $R$ . It follows that all workers with productivity  $x \in [R_0, x^*]$  are better-off as unemployed with  $R = 1$  than they are by implementing any  $R \neq 1$ , for any status quo  $R_0$ .

Finally, by Lemma 1, all the workers who are employed in firms with productivity  $x$  such that  $x > x^*$  strictly prefer to implement  $R = 1$  to any  $R \neq 1$ , for any value of  $R_0$  if their job is not destroyed due to the reform. If  $\beta \leq \hat{\beta}$ , we have that  $x^* \leq 1$ , which implies that all workers who are employed in the status quo in firms with productivity  $x > x^*$  remain employed for any  $R \in [R_0, 1]$ .

We conclude that  $R = 1$  defeats *at unanimity* in pairwise comparison any possible alternative value of  $R$ , and therefore it is the unique Condorcet winner emerging from a majority voting process, whatever the status quo is.

## 8.7 Proof of Lemma 4

We remind that  $\bar{x}$  is defined implicitly by equation (27), setting  $R_0 = \underline{R}$  and  $R' = 1$ . That  $\partial \bar{x} / \partial \beta < 0$  follows immediately from the fact that the left-hand-side of equation (27) is increasing in  $\bar{x}$  while the right-hand-side of the same equation is decreasing in  $\beta$ . Also, equation (27) implies that  $\bar{x}$  depends on  $\sigma$  and  $\mu$  only through  $\alpha$ . By differentiating implicitly  $\bar{x}$  in equation (27) with respect to  $\alpha$ , we obtain that

$$\frac{\partial \bar{x}}{\partial \alpha} = \frac{\bar{x} \left\{ \Delta^{\frac{1-\beta}{\beta}} + \alpha^2 (\underline{R})^{1-\alpha} \bar{x}^\alpha [\ln(\bar{x}) - \ln(\underline{R})] \right\}}{\alpha^2 [\bar{x} - \alpha (\underline{R})^{1-\alpha} \bar{x}^\alpha]}.$$

where  $\Delta \equiv 1 - (\underline{R})^{1-\alpha}$ . The sign of this expression is positive since both the numerator and the denominator are obviously positive. The proof of Lemma 4 follows from how  $\alpha$  depends on  $\sigma$  and  $\mu$  (see (44)).

## 8.8 Proof of Proposition 3

The proof is articulated in two parts. We first show that the possible outcome of a majority voting process are only three, i.e. the status quo  $R_0$ ,  $\underline{R}$  and 1. Then we show that the social preference relation induced by majority voting, denoted as  $\succ$ , is transitive (i.e. there are no Condorcet cycles). We use this preliminary result to demonstrate the political equilibrium always exists, and it is either  $R = 1$  or  $R = \underline{R}$ .

We begin by stating the following two preliminary results.



**Claim 1** Suppose that  $\exists R = R' < R_0$  such that  $R' \succ R_0$ , then  $\underline{R}$  (i.e. maximum rigidity) defeats any  $R$  such that  $R \leq R'$  in pairwise comparisons.

**Proof.** Lemma 1 implies that  $R'$  is preferred to  $R_0$  by all and only the employed workers with productivity above  $x^*$ . Also, the value of these workers  $\mathcal{W}(\cdot | R)$  is strictly increasing in  $R$ , for any  $R > R_0$ . In addition, all the unemployed prefer  $R'$  since their value  $\mathcal{U}$  is strictly increasing in  $R$  for any  $R$ . It follows that if a majority of workers prefer  $R'$  to  $R_0$ , then the same majority of workers prefer  $\underline{R}$  to any  $R$  such that  $R \leq R'$ . ■

**Claim 2** Suppose that  $\exists R = R'' > R_0$  such that  $R'' \succ R_0$ , then  $R = 1$  (i.e. maximum flexibility) defeats any  $R$  such that  $R \geq R''$  in pairwise comparisons.

**Proof.** Lemma 1 and Lemma 2 imply that  $R''$  is preferred to  $R_0$  by all the employed workers with productivity above  $x^*$  and with productivity  $x \in [R_0, x_0'']$  where  $x_0''$  is defined, similarly to  $x_0'$  in Lemma 2, as the level of productivity such that  $\mathcal{W}(x_0'' | R_0) = \mathcal{U}(R'')$ . Also, the value of the employed workers with productivity  $x$  above  $x^*$ ,  $\mathcal{W}(x | R'')$ , is strictly increasing in  $R''$ , and the value that the employed workers with productivity  $x \in [R_0, x_0'']$  obtain from the reform,  $\mathcal{U}(R'')$ , is strictly increasing in  $R''$ . It follows that both these set of workers, and the workers who are unemployed in the status quo, prefer  $R = 1$  to any  $R$  such that  $R \geq R''$ . ■

Claim 1 and Claim 2 imply that, in order to characterize the political equilibrium of the model, we can restrict the attention to the choice between three possible levels of reservation productivity, the status quo  $R_0$ ,  $\underline{R}$  and  $R = 1$ , since any other value of  $R$  is defeated in pairwise comparisons by at least one of these alternatives, i.e. it is not a Condorcet winner.

The rest of the proof leads to the demonstration of existence of a political equilibrium and to its characterization. We consider separately the outcomes of the voting over  $R$  in two possible cases contemplated by Proposition 3, of  $R_0 < \bar{x}$  and of  $R_0 \geq \bar{x}$  respectively, where we remind that  $\bar{x}$  is defined as unique productivity level such that  $\mathcal{W}(\bar{x} | \underline{R}) = \mathcal{U}(1)$ .

**Case 1**  $R_0 < \bar{x}$ .

We rely on the following preliminary result.

**Claim 3** Let  $\bar{x}_0$  be defined as the unique productivity level such that  $\mathcal{W}(\bar{x}_0 | R_0) = \mathcal{U}(1)$ ; we have that  $\bar{x} < \bar{x}_0$  if  $R_0 > \underline{R}$ , and  $\bar{x} = \bar{x}_0$  if  $R_0 = \underline{R}$ .

**Proof.** Consider the equation  $\mathcal{W}(x|R) = \mathcal{U}(1)$ , where as usual  $\mathcal{W}(x|R)$  is defined in (25). Differentiating implicitly this equation with respect to  $R$ , we obtain that

$$\frac{\partial x}{\partial R} = -\frac{(1-\alpha)}{\phi} \frac{1-x^\alpha \phi}{R^\alpha + \alpha^+ R x^{1-\alpha}},$$

where  $\phi \equiv \beta\alpha^+/(1-\beta)$ . The denominator of this expression is positive, and therefore the sign of  $\partial x/\partial R$  is positive if  $(1-x^\alpha \phi) < 0$ . A simple manipulation of this expression shows that  $\partial x/\partial R > 0$  if  $x < x^*$ , where we remind that  $x^*$  is defined by (26). We already know (see Remark 3) that if  $\beta > \hat{\beta}$ , then  $\bar{x} < x^*$ ; it is straightforward to use the same argument made in Remark 3 to conclude that  $\bar{x}_0 < x^*$  if  $\beta > \hat{\beta}$ . Since  $\max\langle \bar{x}, \bar{x}_0 \rangle < x^*$ , we have that  $\partial x/\partial R > 0$  for  $x \in \{\bar{x}, \bar{x}_0\}$  and, because  $R_0 > \underline{R}$ , this implies that  $\bar{x}_0 > \bar{x}$ . The second statement of the claim is obvious. ■

We know from Lemma 1 that  $\underline{R}$  is preferred strictly to  $R_0$  by the employed with productivity  $x \in (R_0, x^*)$ . This set has Lebesgue-Stieltjes (henceforth LS) measure<sup>28</sup>

$$\lambda_0 \equiv \lambda_\Psi \{(R_0, x^*)\} L.$$

Moreover, according to Lemma 2,  $R_0$  is preferred to  $R = 1$  by the workers with productivity  $x \in (\bar{x}_0, x^*)$ , which has LS measure

$$\lambda_1 \equiv \lambda_\Psi \{(\bar{x}_0, x^*)\} L.$$

Finally,  $\underline{R}$  is preferred to  $R = 1$  by the workers with productivity  $x \in (\bar{x}, x^*)$ , which has LS measure

$$\lambda_2 \equiv \lambda_\Psi \{(\bar{x}, x^*)\} L.$$

If  $R_0 > \underline{R}$ , Claim 3 implies that  $\bar{x}_0 < \bar{x}$  implies that  $(\bar{x}_0, x^*) \subset (\bar{x}, x^*)$ , we have that the following inequalities hold

$$\lambda_1 < \lambda_2 < \lambda_0. \tag{76}$$

We distinguish two sub-cases, 1A and 1B, depending on the value of  $\lambda_0$ .

Sub-case 1A:  $\lambda_0 \leq 1/2$ . In this case,  $\lambda_1$  and  $\lambda_2$  and also both lower than  $1/2$  by (76). This means that  $R_0 \succ \underline{R}$ ,  $1 \succ R_0$  and  $1 \succ \underline{R} \Rightarrow R = 1$  defeats any alternative.

Sub-case 1B:  $\lambda_0 > 1/2$ . We have to consider three possibilities. (1)  $\lambda_1$  and  $\lambda_2$  are also both greater than  $1/2$ . In this case, we have that  $\underline{R} \succ R_0$ ,  $R_0 \succ 1$  and  $\underline{R} \succ 1 \Rightarrow \underline{R}$  defeats

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<sup>28</sup>We remind that  $\Psi(\cdot)$  is defined as a distribution function across employment, and thus we need to multiply  $\lambda_\Psi$  by  $L$  to compute the size of the coalition in question.

any alternative. (2)  $\lambda_1 < 1/2$  and  $\lambda_2 \geq 1/2$ . In this case we have that  $\underline{R} \succ R_0$ ,  $1 \succ R_0$ , and  $\underline{R} \succ 1 \Rightarrow \underline{R}$  defeats any alternative. (3)  $\lambda_1$  and  $\lambda_2$  are both lower than  $1/2$ . In this case, we have that  $\underline{R} \succ R_0$ ,  $1 \succ R_0$ , and  $1 \succ \underline{R} \Rightarrow R = 1$  defeats any alternative.

We conclude that the social preference relation  $\succ$  induced by majority voting is transitive, i.e. a conditional political equilibrium exists. Moreover,  $R = 1$  defeats any alternative in pairwise comparisons if  $\lambda_2 \geq 1/2$  and, vice versa,  $R = 1$  defeats any alternative in pairwise comparisons if  $\lambda_2 < 1/2$ .

If  $R_0 = \underline{R}$ , Claim 3 implies that  $\bar{x}_0 = \bar{x}$ , i.e. that  $\lambda_1 = \lambda_2$ . It is straightforward to deduce that the political equilibrium exists also in this special case, and it is the same as described above.

**Case 2**  $R_0 \geq \bar{x}$ .

In this case, we have that  $\lambda_2 = \lambda_0$ . The proof is almost identical to the one relative to that of Case 1 and it is therefore omitted.

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